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# ON THE GENERALIZED PID CONTROL OF LINEAR DYNAMIC SYSTEMS

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## Abstract

In this article we present a tutorial introduction to Generalized Proportional-Integral-Derivative Control (GPID). We particularly emphasize the links between flatness and GPID control, in the context of linear dynamic systems. We present several illustrative design examples, some of them of physical flavor. The performance of the controlled systems is evaluated by means of digital computer simulations.

## 1 Introduction

One of the main drawbacks of modern state space-based feedback control of dynamic systems is constituted by the need to completely measure the state of the system or to estimate it, by means of either asymptotic observers or, in practise, to resort to calculations based on time discretizations of the measured signals. Either approach reduces, in an important manner, the performance of the preferred feedback control scheme with respect to unmodelled parameter variations and other external perturbations. For the continuous regulation of linear systems, the need for state observers, or time discretizations, has been recently side-stepped by the introduction of a new design approach called "Generalized PID Control" (GPID) (see Fliess [1], Fliess *et al* [4]). The theoretical foundations of this technique are found in the concept of localizations, within the context of the module theoretic approach for linear systems. The GPID technique is an integrated estimation-feedback controller design based on "structural" estimates of the state variables rather than on their asymptotic estimates. The structural estimates are formed using inputs, outputs and iterated integrals of the inputs and the output signals of the system. Thus, they can be easily synthesized by use of, either, digital microprocessors or traditional analog electronic circuits. Evidently, such "integral reconstructors" exhibit constant "off-sets" and iterated integrals of such off-sets when compared with the actual state values. Nevertheless, one proceeds to complete the state based closed loop system design by complementing the use of the structural estimates of the state variables with the use of additional, robustifying, integral control actions. The added integral control action is based, naturally, on the output errors and they are devoted to adequately compensate for the de-stabilizing effect of the incurred state estimation errors. In essence, GPID control is an improved output feedback control technique without any of its inherent limitations. It indirectly combines the full power of state feedback, without devoting any efforts to exact state reconstruction, with the known benefits of integral control. The GPID feedback control technique has been already used in the effective regulation of an experimental DC motor system platform with excellent results (See Marquez *et al*,[5]).

In section 2 we briefly revisit GPID control from an elementary viewpoint. In particular, we stress the importance of the flatness property (See Fliess *et al* [2], [3] for basic definitions) in the state feedback controller design task for either stabilization or output trajectory tracking objectives. This issue is particularly important in output reference trajectory tracking tasks for non-minimum phase systems, where the output reference tracking problem must be translated into a flat output reference trajectory tracking task. The flatness based controller is then synthesized in terms of integral input-output parameterizations of the system's state variables. Section 3 is devoted to explore the implications of the design approach in the regulation of some nonlinear systems of physical nature. We illustrate the obtained results by means of computer simulations.

## 2 Some illustrative examples

### 2.1 Controlling a second order integrator

Consider the second order system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \\ y &= x_1 \end{aligned} \tag{1}$$

The system is evidently flat, and also observable, hence, constructible. The flat output is just the system output $y$ which we also denote by $F$ Assume that the initial value of the non measured state $x_2$ is completely unknown and denote it by by $x_{20}$. We have then the following differential parameterization of the system variables in terms of the flat output:

$$\begin{aligned} x_1 &= y = F \\ x_2 &= \dot{y} = \dot{F} \\ u &= \ddot{y} = \ddot{F} \end{aligned} \tag{2}$$

The integral input output parameterization of the system state variables is given, modulo initial conditions, by

$$\begin{aligned} \hat{x}_2 &= \int_0^t u(\tau)d\tau \\ \hat{x}_1 &= x_1 = y \end{aligned} \tag{3}$$

Note the exact relationship between the previous integral parameterization and the system state variables:

$$\begin{aligned} x_2 &= \int_0^t u(\tau)d\tau + x_{20} = \hat{x}_2 + x_{20} \\ \hat{x}_1 &= y \end{aligned} \tag{4}$$

A stabilizing linear state feedback control law for the given linear system may be proposed to be

$$u = -k_1 x_1 - k_2 x_2 = -k_1 y - k_2 \dot{y} = -k_1 F - k_2 \dot{F} \tag{5}$$

with $k_1$ and $k_2$ being strictly positive constant coefficients as of yet unspecified value. Let $\xi$ be an auxiliary control input. In order to avoid the measurement of the (flat) output derivative, $\dot{y}$, we use instead of (5) the following control law complemented with an auxiliary input variable $\xi$ yet to be determined,

$$u = -k_1 y - k_2 \hat{x}_2 + \xi = -k_1 y - k_2 \int_0^t u(\tau)d\tau + \xi \tag{6}$$

Since the variable $\hat{x}_2$ has an offset error with respect to $x_2$, it can be easily seen that this feedback law, in terms of the actual states $x_1$ and $x_2$ and the unknown initial state value $x_{20}$, is equivalently given by

$$u = -k_1 x_1 - k_2 x_2 + k_2 x_{20} + \xi \tag{7}$$

The purpose of the auxiliary input variable $\xi$ should now be clear, as it must compensate for the disregarded initial value of the unavailable variable $x_2$. The closed loop system, using the control law $\hat{u}$ is given by

$$\ddot{y} + k_2 \dot{y} + k_1 y = k_2 x_{20} + \xi \tag{8}$$

Letting $\dot{\xi} = -k_0 y$ with $\xi(0) = 0$, one obtains the closed loop system

$$\ddot{y} + k_2 \dot{y} + k_1 y + k_0 \int_0^t y(\tau)d\tau = k_2 x_{20} \tag{9}$$

Evidently, for any initial condition $x_{20}$, the closed loop system dynamics satisfies the relation:

$$y^{(3)} + k_2 \ddot{y} + k_1 \dot{y} + k_0 y = 0 \tag{10}$$

which can be made asymptotically exponentially stable by suitable choice of the constant coefficients $k_0$, $k_1$ and $k_2$.

We summarize the obtained controller in the following proposition.

**Proposition 2.1** *Let $k_0$,$k_1$ and $k_2$ be constant coefficients of the Hurwitz polynomial $p(s) = s^3 + k_2 s^2 + k_1 s + k_0$. Then, given the linear system (1), the following dynamic feedback control law, synthesized purely in terms the output, the input, and some of their integrals,*

$$\begin{aligned} u &= -k_1 y - k_2 \int_0^t u(\tau)d\tau + \xi \\ \dot{\xi} &= -k_0 y, \quad \xi(0) = 0 \end{aligned} \tag{11}$$

*renders the closed loop system exponentially asymptotically stable as it is governed by*

$$y^{(3)} + k_2 \ddot{y} + k_1 \dot{y} + k_0 y = 0 \tag{12}$$

Figure 1 shows the GPID feedback controller scheme for the second order integrator. Figure 2 shows the simulation of the closed loop system response and the various signals. We used the following strictly positive design parameters:

$$k_0 = \omega_n^2 \beta, \ k_1 = 2\xi\omega_n\beta + \omega_n^2, \ k_2 = 2\xi\omega_n + \beta$$

which yield a characteristic polynomial for the closed loop system of the form $(s^2 + 2\xi\omega_n s + \omega_n^2)(s + \beta)$, with

$$\xi = 0.707, \ \omega_n = 3, \ \beta = 2$$

In figure 1 we also show the structural estimate of $x_2 = \dot{y}$ which exhibits a constant off-set with respect to the actual value.

## 3 GPID Control of some physical systems

### 3.1 Stabilizing a boost converter circuit without inductor current measurements

The average normalized model of a PWM controlled boost converter circuit is given by

$$\begin{aligned} \dot{x}_1 &= -u x_2 + 1 \\ \dot{x}_2 &= u x_1 - x_2/Q \\ y &= x_2 \end{aligned} \tag{13}$$

yields a closed loop characteristic polynomial for the flat output behavior of the form $(s^2 + 2\xi\omega_n s + \omega_n^2)(s + \beta)$.

We summarize the previous result by means of a proposition.

**Proposition 3.1** *The average model of the nonlinear "boost" converter circuit is locally asymptotically exponentially regulated to a desired average output voltage equilibrium value, $\overline{y}$, by means of the following GPID feedback controller:*

$$\begin{aligned} u &= \overline{u} + u_\delta \\ u_\delta &= R y_\delta(t) + \theta \\ \dot{\theta} &= S y_\delta + P u_\delta, \ \ \theta(0) = 0 \end{aligned} \qquad (20)$$

*with the constant parameteres $P$, $R$ and $S$ given by,*

$$\begin{aligned} S &= \frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}\left[\frac{\overline{y}}{Q}(\frac{1}{Q} - k_2) + \frac{1}{\overline{y}}(\frac{1}{\overline{y}^2} - k_1)\right] \\ P &= \frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}\left[\frac{\overline{y}^3}{Q}(\frac{1}{Q} - k_2) + \overline{y}(\frac{1}{\overline{y}^2} - k_1)\right] \\ R &= -\frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}\left[\overline{y}(\frac{1}{Q} - k_2) - \frac{2\overline{y}}{Q}(\frac{1}{\overline{y}^2} - k_1) - \alpha\right] \end{aligned} \qquad (21)$$

*The design parameters $k_1$, $k_2$ and $\alpha$ are chosen so that the following polynomial in the complex variable $s$ is Hurwitz.*

$$p(s) = s^3 + k_2 s^2 + (k_1 - \alpha\frac{Q\overline{y}}{Q^2 + 2\overline{y}^2})s + \alpha(\frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)})$$

As clearly portrayed in Figure 3, the proposed dynamic feedback controller only processes the incremental output voltage $y_\delta$ and the incremental control input $u_\delta$, with no measurements of the average inductor current $x_1$.

Figure 4 shows the GPID feedback controlled responses of the normalized nonlinear boost converter (13) for small initial state perturbations around the desired equilibrium point.

In order to test the robustness of the GPID control scheme, we hypothesize an unmodelled sudden (i.e. discontinuous) variation of the normalized load parameter value $Q$. The variation was allowed to be of 500% the nominal value, occurring at normalized time $t = 10$. This was followed by a *permanent* variation of $Q$, at time $t = 25$, to one fifth of its original value. Although, as expected, the inductor current is severely affected by these variations, the output voltage automatically recovers the desired constant reference value. The simulation results in Figure 5 clearly demonstrates the remarkable robustness of the proposed approach.

### 3.2 Control of the cart-pole system measuring only the cart position

Consider the cart-pole system, shown in Figure 6. This system has been extensively treated in the literature as an example of a nonlinear system which is unstable and non-minimum phase. It is required to control the system, through the application of an external force, so that the pendulum rests on its unstable vertical equilibrium position. The nonlinear model of the system, neglecting friction terms, is given by

$$\begin{aligned} mL\cos\theta\ddot{x} + (J + mL^2)\ddot{\theta} &= mLg\sin\theta \\ (M+m)\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta &= f \end{aligned} \qquad (22)$$

where $x$ is the horizontal distance traveled by the cart and $\theta$ is the angle of the pendulum with respect to the vertical line. $M$ and $m$ stand, respectively, for the cart and the pendulum masses, $L$ is the distance from the base pivot to the center of mass of the pendulum and $g$ is the gravity acceleration. $J$ stands for the pendulum's moment of inertia with respect to the center of gravity. The control input is the force $f$, applied to the cart. We assume that only the variable $x$ may be measured.

The nonlinear system (22) is known to be non-differentially flat. The system's Jacobian linearization around the unstable equilibrium point, located at the value $x = \overline{x} = constant$ and $\theta = \overline{\theta} = 0$, is characterized by the incremental state $[x_\delta, \dot{x}_\delta, \theta_\delta, \dot{\theta}_\delta]$, defined as

$$x_\delta = x - \overline{x}, \ \ \dot{x}_\delta = \dot{x}, \ \ \theta_\delta = \theta, \ \ \dot{\theta}_\delta = \dot{\theta}, \ \ f_\delta = f$$

The linearized system, written in normalized variables: $\xi_\delta = (x - \overline{x})/L, u_\delta = f/mg$ and $\tau = (\sqrt{g/L})\ t$, is given by

$$\begin{aligned} \ddot{\xi}_\delta + \epsilon\ddot{\theta}_\delta &= \theta_\delta \\ \gamma\ddot{\xi}_\delta + \ddot{\theta}_\delta &= u_\delta \end{aligned} \qquad (23)$$

with $\epsilon = (1 + J/mL^2), \gamma = (1 + M/m)$.

The linearized system (23) is, however, controllable and hence flat. Moreover, the system is observable from the normalized incremental position variable $\xi_\delta$ and it is *unobservable* from the angular position variable $\theta_\delta$. The flat output is given by,

$$F_\delta = \xi_\delta + \epsilon\theta_\delta$$

which has the physical interpretation of being the normalized value of the incremental center of oscillation of the system (the Huygen's center of oscillation). The natural state variables of the linearized system and the incremental control input may be expressed in terms of $F_\delta$ and a finite number of its time derivatives,

$$\begin{aligned} \xi_\delta &= F_\delta - \epsilon\ddot{F}_\delta, \ \ \dot{\xi}_\delta = \dot{F}_\delta - \epsilon F_\delta^{(3)} \\ \theta_\delta &= \ddot{F}_\delta, \ \ \dot{\theta}_\delta = F_\delta^{(3)} \\ u_\delta &= (1 - \epsilon\gamma)F_\delta^{(4)} + \gamma\ddot{F}_\delta \end{aligned} \qquad (24)$$

Notice that if we regulate the incremental flat output $F_\delta$ to zero, along with its time derivatives, then the incremental normalized distance $\xi_\delta$ and the angular position, $\theta_\delta = \theta$, are both, simultaneously, regulated to the desired equilibrium.

A stabilizing flatness-based controller is immediately obtained from the previous parameterization. This is given by

$$u_\delta = \gamma\ddot{F}_\delta + (1-\epsilon\gamma)\left[-k_5 F_\delta^{(3)} - k_4\ddot{F}_\delta - k_3\dot{F}_\delta - k_2 F_\delta\right] \qquad (25)$$

where $u$ is the average control input (also known as "duty ratio") bounded within the interval $[0,1]$. The state variables $x_1$ and $x_2$ represent, respectively, the average inductor current and the average capacitor voltage which is the system output. The parameter $Q$ is the circuit quality, directly related to the value of the load resistance. The constant additive input "1" represents the normalized constant voltage source value.

The capacitor voltage is a non-minimum phase output variable as it has been shown in Sira-Ramírez and Lischinsky-Arenas in [7]. It is usually required to keep this output voltage at a fixed desired, strictly positive, average equilibrium value $\overline{x}_2 = \overline{y} > 1$. The corresponding equilibrium values of the inductor current and the control input are given by

$$\overline{x}_1 = \frac{\overline{y}^2}{Q}, \quad \overline{u} = \frac{1}{\overline{y}}$$

The jacobian linearization of the system, around the operating equilibrium point, is given by

$$\begin{aligned} \dot{x}_{1\delta} &= -\frac{1}{\overline{y}} x_{2\delta} - \overline{y} u_\delta \\ \dot{x}_{2\delta} &= \frac{1}{\overline{y}} x_{1\delta} - \frac{1}{Q} x_{2\delta} + \frac{\overline{y}^2}{Q} u_\delta \\ y_\delta &= x_{2\delta} \end{aligned} \tag{14}$$

where $x_{1\delta} = x_1 - \overline{x}_1$, $x_{2\delta} = x_2 - \overline{y}$ and $u_\delta = u - \overline{u}$ are the incremental variables around the given equilibrium.

The linearized system transfer function clearly depicts the non-minimum phase character of the system incremental output

$$y_\delta = \frac{\overline{y}^2}{Q} \left( \frac{s - Q/\overline{y}^2}{s^2 + (1/Q)s + (1/\overline{y}^2)} \right) u_\delta \tag{15}$$

The linearized system is controllable and observable, hence flat and constructible. The flat output and its time derivative are given by the expressions

$$\begin{aligned} F_\delta &= \overline{x}_1 x_{1\delta} + \overline{x}_2 x_{2\delta} = \left(\frac{\overline{y}^2}{Q}\right) x_{1\delta} + \overline{y} x_{2\delta} \\ \dot{F}_\delta &= x_{1\delta} - \frac{2\overline{y}}{Q} x_{2\delta} \end{aligned} \tag{16}$$

which are, respectively, the incremental average total stored energy and the incremental instantaneous average consumed power. The differential parameterization of the state variables is given by

$$\begin{aligned} x_{1\delta} &= \frac{2Q}{Q^2 + 2\overline{y}^2}\left(F_\delta + \frac{Q}{2}\dot{F}_\delta\right) \\ x_{2\delta} &= \frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}\left(F_\delta - \frac{\overline{y}^2}{Q}\dot{F}_\delta\right) \\ u_\delta &= \frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}\left(\ddot{F}_\delta + \frac{1}{Q}\dot{F}_\delta + \frac{1}{\overline{y}^2}F_\delta\right) \end{aligned}$$

The integral input output parameterization of the incremental average state variables and that of the flat output are readily obtained from the system equations and the definition of the flat output, as

$$\begin{aligned} \widehat{x}_{1\delta} &= -\frac{1}{\overline{y}} \int_0^t y_\delta(\tau)d\tau - \overline{y} \int_0^t u_\delta(\tau)d\tau \\ \widehat{x}_{2\delta} &= y_\delta \\ \widehat{F}_\delta &= \overline{y} y_\delta - \frac{\overline{y}}{Q} \int_0^t y_\delta(\tau)d\tau - \frac{\overline{y}^3}{Q} \int_0^t u_\delta(\tau)d\tau \\ \widehat{\dot{F}}_\delta &= -\frac{1}{\overline{y}} \int_0^t y_\delta(\tau)d\tau - \overline{y} \int_0^t u_\delta(\tau)d\tau - \frac{2\overline{y}}{Q} y_\delta \end{aligned}$$

Note that the actual relation linking the incremental flat output and its time derivative to their estimates is given by

$$F_\delta = \widehat{F}_\delta + \frac{\overline{y}^2}{Q} x_{1\delta}(0), \quad \dot{F}_\delta = \widehat{\dot{F}}_\delta + x_{1\delta}(0) \tag{17}$$

where $x_{1\delta}(0)$ is the unknown initial value of the average incremental current variable $x_1$. As in many other instances, the previous relation reveals that: "the derivative of the estimate does not necessarily coincide with the estimate of the derivative".

A flatness based stabilizing incremental feedback controller is immediately obtained from the differential parameterization of the incremental input variable $u_\delta$ as

$$u_\delta = -\frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)} \left[ (\frac{1}{Q} - k_2)\dot{F}_\delta + (\frac{1}{\overline{y}^2} - k_1) F_\delta \right] \tag{18}$$

Based on the previous controller, we proceed to propose the GPID controller as

$$\begin{aligned} u_\delta &= -\frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)} \left[ (\frac{1}{Q} - k_2)\widehat{\dot{F}}_\delta + (\frac{1}{\overline{y}^2} - k_1)\widehat{F}_\delta + \xi \right] \\ \dot{\xi} &= -\alpha y_\delta \end{aligned}$$

The closed loop dynamics, after defining $\xi$ by $\dot{\xi} = -\alpha y_\delta$, is seen to be governed by

$$\begin{aligned} \ddot{F}_\delta &= -\left[ \left(\frac{1}{Q} - k_2\right) + \left(\frac{1}{\overline{y}^2} - k_1\right) \frac{\overline{y}^2}{Q} \right] x_{1\delta}(0) + \xi \\ &\quad - k_2 \dot{F}_\delta - k_1 F_\delta \\ \dot{\xi} &= -\alpha y_\delta \end{aligned}$$

which is equivalent to

$$F_\delta^{(3)} + k_2 \ddot{F}_\delta + (k_1 - \alpha \frac{Q\overline{y}}{Q^2 + 2\overline{y}^2})\dot{F}_\delta + \alpha (\frac{Q^2}{\overline{y}\,(Q^2 + 2\overline{y}^2)}) F_\delta = 0 \tag{19}$$

The system is seen to be rendered asymptotically exponentially stable for a suitable choice of the design parameters $k_1$, $k_2$ and $\alpha$. For instance, the choice

$$\alpha = \frac{\overline{y}(Q^2 + 2\overline{y}^2)}{Q^2} \omega_n^2 \beta, \quad k_1 = \frac{\overline{y}^2}{Q} + \omega_n^2 + 2\xi\omega_n\beta,$$

$$k_2 = 2\xi\omega_n + \beta$$

with $k_5, .., k_2$ being design constants to be determined later.

Consider the output $y_\delta = \xi_\delta$. An integral input-output parameterization of the flat output and its time derivatives is readily obtained as

$$\begin{aligned} \widehat{F_\delta} &= (1-\epsilon\gamma)y_\delta + \epsilon(\int\int u_\delta) \\ \widehat{\dot{F}_\delta} &= (\int\int\int u_\delta) - \gamma(\int y_\delta) \\ \widehat{\ddot{F}_\delta} &= (\int\int u_\delta) - \gamma y_\delta \\ \widehat{F_\delta^{(3)}} &= \frac{1}{1-\epsilon\gamma}\left[(\int u_\delta) - \gamma(\int\int\int u_\delta) + \gamma^2(\int y_\delta)\right] \end{aligned} \tag{26}$$

The above structural estimates can be used in the following GPID controller

$$\begin{aligned} u &= u_\delta = \gamma\widehat{\ddot{F}_\delta} + (1-\epsilon\gamma)\Big[-k_5\widehat{F_\delta^{(3)}} - k_4\widehat{\ddot{F}_\delta} - k_3\widehat{\dot{F}_\delta} \\ &\quad -k_2\widehat{F_\delta} + \xi\Big] \\ \dot{\xi} &= -\alpha y_\delta + \eta, \quad \xi(0) = 0 \\ \dot{\eta} &= -\beta y_\delta, \quad \eta(0) = 0 \end{aligned} \tag{27}$$

which yields the following closed loop dynamics for the flat output:

$$\begin{aligned} &F_\delta^{(6)} + k_5 F_\delta^{(5)} + k_4 F\delta^{(4)} + (k_3 - \alpha\epsilon)F_\delta^{(3)} \\ &+(k_2 - \beta\epsilon)\ddot{F}_\delta + \alpha\dot{F}_\delta + \beta F_\delta = 0 \end{aligned}$$

Figure 7 depicts the time responses of the closed loop nonlinear cart-pole system (22) to a small initial angular deviation. The system parameters were set to be

$$M = 1.5\ \mathrm{Kg},\quad m = 0.3\ \mathrm{Kg},\quad g = 9.8\ \mathrm{m/s^2},$$

$$L = 0.3,\ \mathrm{m},\quad J = \frac{1}{3}mL^3$$

The coefficients for the controller were chosen so that the closed loop characteristic polynomial for the flat output coincided with: $(s^2 + 2\xi\omega_n s + \omega_n^2)^3$. We set: $\xi = 0.88, \omega_n = 0.80$.

## 4 Conclusions

In this article, we have given an elementary introduction to GPID control of linear systems. The GPID approach avoids the need for asymptotic state observers, or time discretizations, in order to estimate the required state variables in any given state feedback control scheme. This is valid whether the controller is based on the flatness of the system or not. The emphasis in GPID is placed in obtaining "structural", rather than *exact* estimates of the states. The structural estimates are given purely in terms of outputs and iterated integrals of the outputs and of the inputs. This program is certainly possible for linear systems which are observable and, hence, constructible. Roughly speaking, constructibility means, that the state can be expressed in terms of inputs and outputs and a finite number of their time derivatives. This fact can be also used, with the help of the state equations, to effectively obtain an integral input-output parameterization of the system states, modulo initial conditions and iterated integrals of such initial conditions. The effect of the state estimation errors, due to the open loop nature of the underlying observers, in the closed loop dynamics is then compensated by use of appropriate integral control action based on iterated integrals of the output error and possibly, the input error. The approach is rather robust with respect to unmodelled parameter variations as demonstrated in some of the treated examples.

Some extensions already exist to the case of nonlinear systems, (See [8], [9]). The most interesting case of delayed differential systems is treated by Marquez *et al* in [6], from the context of classical Smith predictors. Nevertheless, systematic design procedures for nonlinear systems are still the subject of ongoing research.

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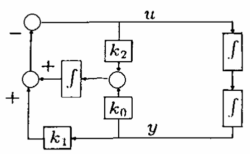


Figure 1: The generalized PID control scheme for a second order integrator

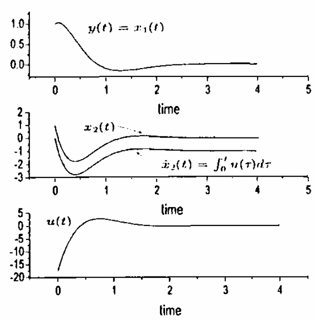


Figure 2: The closed loop performance of a second order integrator generalized PID controlled system

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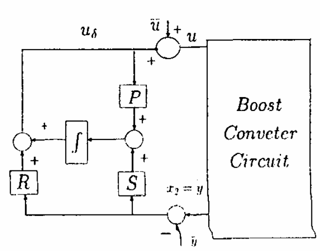


Figure 3: Generalized PID control scheme for the average "boost" converter circuit model

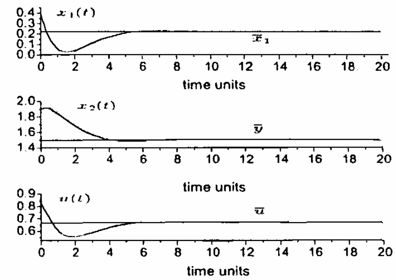


Figure 4: Generalized PID controlled responses for an average "boost" converter circuit model

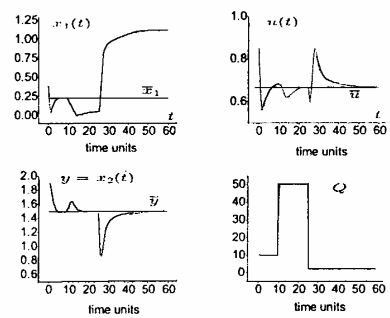


Figure 5: Generalized PID controlled responses of the "boost" converter to large load parameter variations

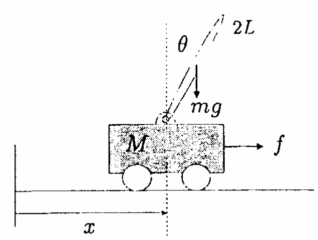


Figure 6: Cart-pole system.

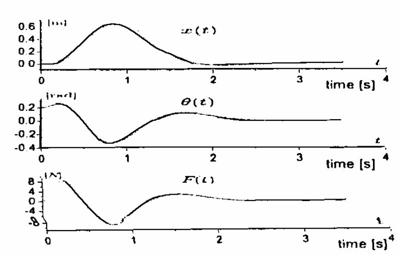


Figure 7: GPID Controlled responses of cart-pole system