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GENERALIZED PI SLIDING MODE CONTROL OF DC-TO-DC POWER CONVERTERS

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Abstract: A sliding mode feedback controller, based on the Generalized PI control approach, is developed for the regulation of the "boost" DC-to-DC power converter circuit. The feedback control scheme uses only output capacitor voltage measurements, as well as knowledge of the available input signal, represented by the switch positions. The robustness of the feedback scheme is tested with abusively large, non-modeled, sudden load resistance variations.

Keywords: Generalized PI Control, Sliding Modes, DC-to-DC Power Converters

1. INTRODUCTION

In this article, we propose a new sliding mode feedback control option for the "boost" converter circuit. The approach is based on the recently introduced idea of *Generalized PI control* (GPI) (See Fliess *et al* 2001, Marquez and Fliess (2000) and Fliess 2000), which elegantly side-steps the need for any observers, or time discretizations, in the feedback regulation of linear dynamic systems. The extension of the inherently linear GPI control technique to the nonlinear scheme of sliding mode control is here accomplished in the context of the regulation of the switched DC/DC power converter circuit of the "boost" type. The basic idea resides in obtaining an integral input-output parameterization of the system state variables, that we address as structural estimates, which allows for the suitable structural estimation of the sliding surface. The sliding surface evolution is then corrected, on line, by means of direct output error integral control action. The GPI sliding mode scheme is shown to share the stabilizing features of the traditional sliding mode control, but it is shown to be vastly superior, as far as robustness with respect to unmodelled parameter variations is concerned. The control scheme is only based on output and input measurements and time integrations of such measurements. This gives traditional "op-amps" and modern integrated ana-

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log circuits a renewed importance in the feedback regulation of power electronics circuits.

Section II presents the boost converter model and establishes the control objectives. It also revisits the rationale behind the traditional sliding mode control approach for the regulation of the boost converter. In section III we introduce the generalized PI sliding mode controller and analytically derive its asymptotically stabilizing properties. Section IV presents some digital computer simulations illustrating the performance of the proposed feedback controller. In this section we also examine the robustness of the proposed feedback control scheme when the circuit is subject to unmodelled, sudden, large load variations (up to 500 % of its nominal value). The obtained results are highly encouraging. Section V is devoted to present the conclusions of this work and gives some suggestions for further research.

2. GENERALIZED PI CONTROL OF THE BOOST CONVERTER

2.1 *The boost converter model*

Consider the boost converter circuit, shown in Figure 1. The system is described by the set of equations

$$\begin{aligned} L\dot{z}_1 &= -uz_2 + E \\ C\dot{z}_2 &= uz_1 - \frac{z_2}{R} \end{aligned} \qquad (1)$$

where z_1 represents the inductor current and z_2 is the output capacitor voltage. The control input u, representing the switch position function, is a discrete-valued signal taking values in the set $\{0, 1\}$. The system parameters are constituted by: L, which is the inductance of the input circuit; C the capacitance of the output filter and R, the output load resistance. The external voltage source has the constant value E.

We introduce the following state normalization and time scale transformation:

$$x_1 = \frac{z_1}{E}\sqrt{\frac{L}{C}}, \quad x_2 = \frac{z_2}{E}, \quad \tau = \frac{t}{\sqrt{LC}} \qquad (2)$$

The normalized model is thus given by:

$$\begin{aligned} \dot{x}_1 &= -ux_2 + 1 \\ \dot{x}_2 &= ux_1 - \frac{x_2}{Q} \\ y &= x_2 \end{aligned} \qquad (3)$$

where now, with an abuse of notation, the " $\cdot$ " represents derivation with respect to the normalized time, τ. The variable x_1 is the normalized inductor current, x_2 is the normalized output voltage and u, still represents the switch position function. The constant system parameters are all comprised now in the circuit "quality" parameter, denoted by Q and given by the strictly positive quantity, $R\sqrt{C/L}$. It is assumed that the only system variable available for measurement is the output capacitor voltage x_2.

The operating normalized equilibrium point for the system can be computed in the following idealized manner: Assume that by means of an infinite frequency discontinuous control input a constant value, $\overline{x}_2 = V$, of the output capacitor voltage and of the inductor current are achieved. To this constant equilibrium state value, it corresponds a *constant equivalent control*, or *average control input*, denoted by u_{eq}, which is obtained from the first equation in (3), as $\overline{u}_{eq} = 1/V$. The corresponding normalized equilibrium value of the inductor current, according to the second equation of (3) is then given by $\overline{x}_1 = V^2/Q$. Notice that since u_{eq} must be bounded by the closed interval $[0, 1]$, then, necessarily, the achievable normalized constant voltages for x_2 are greater than 1.

The normalized boost circuit equations exhibit two important properties which should be remarked. We summarize these properties in the following proposition (See the CDRom Version for a proof)

Proposition 1. In an average sense, the output capacitor voltage variable x_2 is a *non-minimum phase* variable, while the input inductor current is a minimum phase variable.

These two facts have motivated *indirect* feedback control schemes based on inductor current regulation, or, alternatively, stored energy regulation (see Sira-Ramírez et al (1997), and Fliess *et al* (1998).

2.2 *Control objectives and the traditional sliding mode approach*

The control objective consists in sustaining, by means of discontinuous feedback control, an average constant equilibrium value of the normalized output capacitor voltage, x_2, given by the desired value: $\overline{x}_2 = V_d$.

Unfortunately, the traditional sliding mode controller is based on a measurement of the inductor current, or, equivalently, of its normalized value,

x_1. In practise, and within the domain of DC/DC power converters, the inductor current, x_1, is known to be a hard signal to measure precisely. This is due to the high frequency switchings commanding the inductor current time derivative and the *high pass filter* nature of the input circuit. For this reason, a scheme which is based on the non-minimum phase output variable $y = x_2$ is usually preferable. This will be achieved in the Section 3 by invoking the philosophy of Generalized PI control. The poor robustness characteristics, with respect to load resistance variations, of the traditional sliding mode controller in the *indirect* regulation of the output capacitor voltage is a well known result. (See the CD-rom version)

3. A GPI SLIDING MODE BASED CONTROLLER

The normalized system (3) is observable, in an average sense, from the measured normalized output variable $y = x_2$. This is easily verified since the "observability" matrix:

$$\frac{\partial(y, \dot{y})}{\partial x} = \begin{bmatrix} 0 & 1 \\ u & -\dfrac{1}{Q} \end{bmatrix} \tag{4}$$

is rank 2 for all average values of u which are not identically equal to zero. Since the average value of the input, under ideal sliding mode conditions, is $u_{eq} = 1/V_d > 0$, the observability condition is clearly met.

An integral parameterization of the normalized inductor current, $x_1(\tau)$, in the following equation, is directly obtained from the first equation of (3),

$$\begin{aligned} x_1(\tau) &= \int_0^\tau (1 - u(\rho)y(\rho))\, d\rho \\ x_2(\tau) &= y(\tau) \end{aligned} \tag{5}$$

The integral parameterization of x_1 in equation (5) may be considered to be a "structural estimate" of the normalized inductor current x_1 which is "off" by a constant value, represented by the unknown initial condition $x_1(0)$. We denote by $\widehat{x}_1(\tau)$, the estimate of x_1 in equation (5), i.e.

$$\widehat{x_1}(\tau) = \int_0^\tau (1 - u(\rho)y(\rho))\, d\rho \tag{6}$$

It is clear that the true relation linking the estimated value $\widehat{x_1}$ of x_1 to its actual value, is just given by

$$x_1(\tau) = \widehat{x}_1(\tau) + x_1(0) \tag{7}$$

We use the "faulty" estimate (6) of the inductor current x_1 in the sliding mode controller and complement it with an integral control action, computed on the basis of the output voltage stabilization error, $y - V_d$.

Consider then the following GPI sliding mode controller,

$$u = \begin{cases} 1 \text{ for } \widehat{\sigma}(y, u, \xi) > 0 \\ 0 \text{ for } \widehat{\sigma}(y, u, \xi) < 0 \end{cases} \tag{8}$$

$$\widehat{\sigma}(y, u, \xi) = \int_0^\tau (1 - u(\rho)y(\rho))\, d\rho - \frac{V_d^2}{Q} + k_0\xi \tag{9}$$

$$\dot{\xi} = y(\tau) - V_d, \quad \xi(0) = 0 \tag{10}$$

with k_0 a strictly positive design constant to be chosen later.

The modified sliding surface coordinate function, $\widehat{\sigma}$, can also be equivalently written in terms of the, non-measured, actual state x_1 as,

$$\widehat{\sigma}(x_1, \xi) = x_1 - \frac{V_d^2}{Q} - x_1(0) + k_0\xi \tag{11}$$

This expression is found to be useful for our analysis purposes.

The time derivative of any of the two equivalent expressions of the modified sliding surface coordinate function (11), or (9), is given by

$$\dot{\widehat{\sigma}}(y, u, \xi) = 1 - uy + k_0(y - V_d) \tag{12}$$

Note that on $\widehat{\sigma} = 0$ the inductor current, x_1, is given by the expression $x_1 = \frac{V_d^2}{Q} + x_1(0) - k_0\xi$.

The equivalent control, corresponding to the modified sliding surface coordinate function is now given by

$$u_{eq} = \frac{1 + k_0(y - V_d)}{y} \tag{13}$$

A sliding regime locally exists on $\widehat{\sigma}(y, u, \xi) = 0$ whenever the following *intermediacy condition*, $0 < u_{eq} < 1$, is satisfied (see Sira-Ramírez, 1998):

$$0 < 1 + k_0(y - V_d) < y \tag{14}$$

which is equivalent to the following set of inequalities

$$\begin{cases} V_d - \dfrac{1}{k_0} < y < V_d + \dfrac{1-V_d}{1-k_0} \text{ for } k_0 > 1 \\ y > V_d - \min\{\dfrac{1}{k_0}, \dfrac{V_d-1}{1-k_0}\} \text{ for } 0 < k_0 < 1 \end{cases} \tag{15}$$

Thus, the choice of k_0 as a strictly positive constant bounded by the interval:

$$0 < k_0 < \frac{1}{V_d} < 1$$

guarantees the non-empty character of the region where a sliding mode exists. In fact, the smaller the value of k_0 the wider the region of existence of such sliding motion in the space of the output y. Therefore, the values of the regulated capacitor voltage y, under ideal sliding conditions, satisfy the constraint $y > (1 - k_0 V_d)/(1 - k_0)$.

For the reachability of the sliding surface from the origin, suppose the system is initially resting at the zero state, $x_1(0) = 0, x_2(0) = 0, \xi(0) = 0$, then, the initial value of the modified sliding surface is negative, $\widehat{\sigma}(x_1(0), 0) = \widehat{\sigma}(0,0) = -V_d^2/Q < 0$, and the initial value of the product, $\widehat{\sigma}\dot{\widehat{\sigma}}$, is given by:

$$\widehat{\sigma}(0,0)\dot{\widehat{\sigma}}(0,0) = -\frac{V_d^2}{Q}(1 + k_0 V_d)$$

The modified sliding surface $\widehat{\sigma}$, thus, starts increasing towards zero from the given zero initial condition, provided k_0 is chosen within the prescribed interval. The region of existence of a sliding motion,

$$y > (1 - k_0 V_d)/(1 - k_0)$$

, is, therefore, always reachable from the origin by the proposed switched control strategy (10).

The local reachability of the sliding surface, $\widehat{\sigma} = 0$, from an arbitrary initial state value, is established by the well known condition, $\widehat{\sigma}\dot{\widehat{\sigma}} < 0$, to be verified in a local neighborhood of the modified sliding surface. As we have already seen, this neighborhood includes the origin of the state space, which is a common starting point for the DC/DC power converter operation.

Let $\widehat{\sigma} < 0$, then, according to (8), the control is set to $u = 0$. The time derivative of the modified sliding surface coordinate is given by $\dot{\widehat{\sigma}} = 1 + k_0(y - V_d)$. Then for all $y > V_d - 1/k_0$, the time derivative, $\dot{\widehat{\sigma}}$ is positive and the product $\widehat{\sigma}\dot{\widehat{\sigma}}$ is negative. Suppose now that $\widehat{\sigma}$ is positive, then, the control input is given by $u = 1$. The time derivative of the sliding surface coordinate is $\dot{\widehat{\sigma}} = 1 - y + k_0(y - V_d)$. Thus, for all $y > \frac{1-k_0 V_d}{-k_0} = V_d - \frac{V_d - 1}{1-k_0}$ and the product $\widehat{\sigma}\dot{\widehat{\sigma}}$ is, again, negative. We conclude that a sliding regime exists on the modified sliding surface, $\widehat{S} = \{ (y, u, \xi) \mid \sigma(y, u, \xi) = 0\}$, which is locally reachable in finite time, by means of the proposed discontinuous control law (8).

The ideal sliding dynamics, obtained from the *invariance conditions*, $\widehat{\sigma} = 0$, $\dot{\widehat{\sigma}} = 0$, is now obtained as

$$\begin{aligned} \dot{y} &= \frac{1 + k_0(y - V_d)}{y}\left[\frac{V_d^2}{Q} + x_1(0) - k_0\xi\right] - \frac{y}{Q} \\ \dot{\xi} &= y - V_d \end{aligned} \tag{16}$$

The only constant equilibrium point, $(\overline{y}, \overline{\xi})$, of the ideal closed loop sliding dynamics (16) is given by

$$\overline{y} = V_d, \quad \overline{\xi} = \frac{1}{k_0} x_1(0), \tag{17}$$

It remains to be proved that the nature of the stability of the equilibrium point with respect to ideal sliding trajectories starting on the sliding surface $\widehat{S}$. It may be verified that such an equilibrium point is not attractive from every point the sliding surface. Local asymptotic stability of the equilibrium point (17), which is enough for our purposes. can be proved by resorting to tangent linearization of the ideal sliding dynamics.

We summarize the proven result in the following proposition,

Proposition 2. Given a boost converter, represented in normalized form by (3), in which it is desired to bring the only measurable output variable $y = x_2$ to the desired value $V_d > 0$. Assume that the control input u is also available for measurement. Then, the following GPI sliding mode controller, using only input-output, information:

$$\begin{aligned} u &= \begin{cases} 1 \text{ for } \widehat{\sigma}(y,u) > 0 \\ 0 \text{ for } \widehat{\sigma}(y,u) < 0 \end{cases} \\ \widehat{\sigma}(y,u,\xi) &= \int_0^\tau (1 - u(\rho)y(\rho))\, d\rho - \frac{V_d^2}{Q} + k_0\xi \\ \dot{\xi} &= y(\tau) - V_d, \quad \xi(0) = 0, \quad 0 < k_0 < \frac{1}{V_d} \end{aligned} \tag{18}$$

yields a permanent sliding motion on the surface:

$$\widehat{S} = \{ (y, u, \xi) \mid \widehat{\sigma}(y, u, \xi) = 0\}$$

$$= \{ \ (x, \xi) \mid x_1 - \frac{V_d^2}{Q} - x_1(0) + k_0 \xi = 0 \} \quad (19)$$

which is reachable from the origin in finite time. The induced sliding motions on the sliding manifold $\widehat{S}$, ideally, locally asymptotically stabilize the trajectories of the circuit variables x_1, x_2 and ξ towards the equilibrium values:

$$\overline{x}_1 = \frac{V_d^2}{Q}, \quad \overline{x}_2 = V_d, \quad \overline{\xi} = \frac{1}{k_0} x_1(0)$$

where $x_1(0)$ is the unknown initial state of the normalized inductor current variable x_1. The sliding motions exist on $\widehat{S}$ whenever the regulated values of the output y satisfy the inequality:

$$y \ > \ V_d - \frac{V_d - 1}{k_0 - 1} \quad (20)$$

Figure 2 depicts the GPI sliding mode feedback control scheme for the stabilization of the normalized boost converter circuit.

4. SIMULATION RESULTS

Simulations were performed on a typical boost converter circuit with parameter values given by

$$L = 20 \text{ [mH]}, \ C = 20 \ [\mu\text{F}], \ R = 30 \ [\Omega],$$

$$E = 15 \text{ [V]}$$

This parameter values yield a value of Q given by $Q = 0.9486$ and a time normalization factor given by $t = 6.32 \times 10^{-4} \ \tau$.

It was desired to bring the boost converter trajectories from unknown initial conditions (taken to be, for the simulation purposes, $x_1(0) = 0.5$ and $x_2(0) = 0.8$) towards the final desired value of $\overline{z}_2 = 30$ [V], with corresponding $\overline{z}_1 = 2$ [A]. The simulations, shown in Figure 3, depict the performance of the generalized PI sliding mode control scheme on the specified DC/DC boost converter circuit. The underlying sampling frequency was set to be 158.22 [KHz], and the value of the design constant k_0 was set to be $k_0 = 0.1 < 1/Vd = 0.5$.

4.1 *Robustness to load variations*

In order to test the robustness of the proposed GPI sliding mode control scheme, we let the load resistor R undergo a sudden non-modeled and *permanent* variation of 500 % of its nominal value of 30Ω. This variation took place, approximately, at time, $t = 0.0633$ [s], wile the system was not yet stabilized to the desired voltage value. Figure 4 shows the excellent recovering features of the proposed controller to the imposed load variation.

5. CONCLUSIONS

In this article we have extended the Generalized PI control technique to the realm of sliding mode control, within the context of a specific physical example of wide interest in the Power Electronics area. We have proposed an asymptotically stabilizing sliding mode controller which only requires measurements of the non-minimum phase variable of the converter, represented by the output capacitor voltage. The GPI controller is motivated by the usual design of the traditional sliding surface coordinate function in terms of the normalized inductor current variable alone. A structural estimate, exhibiting a constant "off-set" error, of the normalized inductor current variable is synthesized in terms of integrals of nonlinear functions of the available input and output signals. The sliding surface synthesis uses this faulty estimate of the inductor current with a suitable complementation including an integral output error stabilization. The integral input-output parameterized sliding surface is shown to be locally reachable and, once a sliding regime is established, a locally asymptotically stable ideal sliding dynamics is obtained on the sliding manifold which converges to the desired equilibrium values for the normalized circuit variables.

Through computer simulations, the proposed control scheme was shown to be remarkably robust with respect to large load parameter variations of up to 500 %.

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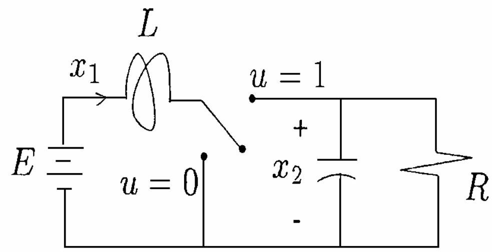

Fig. 1. The boost converter circuit

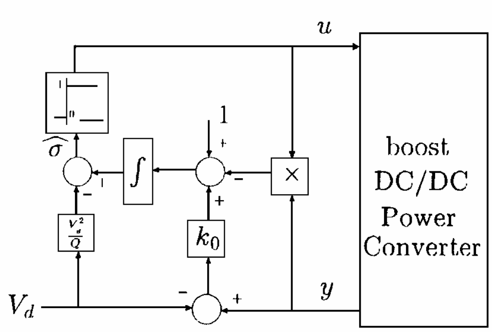

Fig. 2. Generalized PI sliding mode control scheme for the stabilization of the boost converter circuit

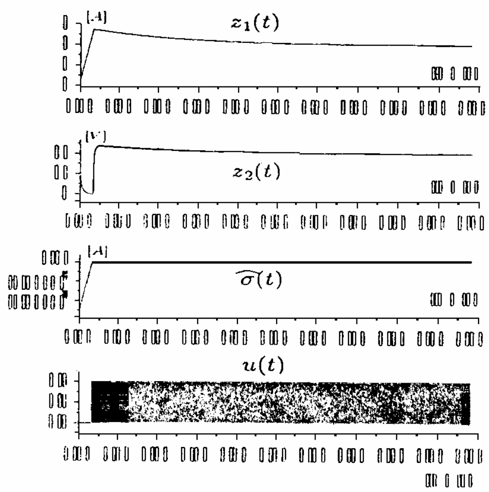

Fig. 3. GPI controlled boost converter performance

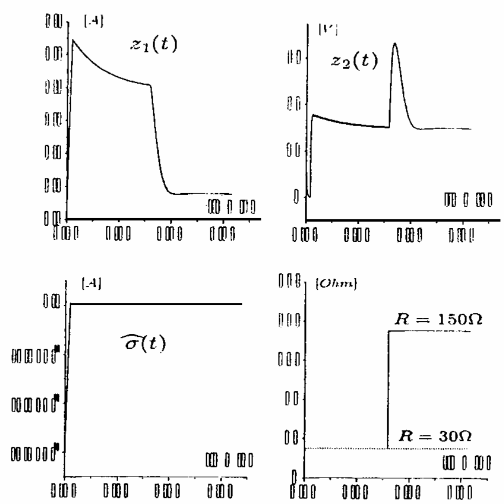

Fig. 4. GPI controlled boost converter performance to unmodelled load variations of 500 %