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Generalized PID control of the Average Boost Converter Circuit Model

Hebertt Sira-Ramírez and Gerardo Silva-Navarro[1]

CINVESTAV-IPN
Dept. Ingeniería Eléctrica, Sección Mecatrónica
Avenida IPN, # 2508, Col. San Pedro Zacatenco, A.P. 14740
07300 México, D.F., México.

Abstract. In this article we examine, in the context of equilibrium-to-equilibrium reference trajectory tracking, the Generalized Proportional Integral Derivative (GPID) control of a nonlinear average model of a DC-to-DC power converter of the "Boost" type. The design approach relies on the converter's tangent linearization model and Lyapunov stability theory. The performance of the feedback controlled nonlinear system is evaluated by means of digital computer simulations including large, unmodelled, load parameter variations.

1 Introduction

Generalized Proportional-Integral-Derivative (GPID) control was introduced by Prof. M. Fliess and his coworkers (See [2] and [3]) in the context of linear time-invariant controllable systems. GPID control sidesteps the need for the traditional asymptotic state observers and directly proceeds to use, in a previously designed state feedback control law, ***structural state estimates*** in place of the actual state variables. These structural estimates are based on *integral reconstructors* requiring only inputs, outputs, and iterated integrals of such available signals. The effect of the neglected initial states is suitably compensated by means of a sufficiently large number of additional iterated integral output error, or integral input error, control actions. The method has enormous interest from a theoretical viewpoint and ties in with the algebraic *module theoretic* approach to linear systems (see Fliess [1]) and the theory of algebraic *localizations* (see also Fliess *et al* ([4])). GPID control can be extended to linear delay systems and it has been extended to particular instances of nonlinear systems in Sira-Ramírez *et al* ([11]).

In this article, we present a study of the relevance of the GPID control for the trajectory tracking in an average nonlinear model of a DC-to-DC power converter of the "Boost" type. Our developments are cast in the context of linearized average models around nominal state reference trajectories accomplishing a desired equilibrium-to-equilibrium transfer. In order to demonstrate the flexibility of the GPID controller implementation approach in accomodating for several state feedback controller design techniques, we use a Lyapunov-based controllers in the state feedback controller design. The performance of the GPID feedback controlled system is evaluated by means of digital computer simulations.

Section 2 deals with the generalities of the average nonlinear model of a Boost converter. We particularly emphasize the flatness of the system and the minimum and non-minimum phase character of its state variables. Section 3 uses a time-varying linearized model of the boost converter, valid around a nominal state and input trajectory, off-line specified on the basis of the nonlinear system flatness. The nominal state trajectory is specified to achieve a typical equilibrium-to-equilibrium transfer taking place in finite time. We implement, via the GPID approach a Lyapunov-based state feedback controller achieving asymptotic stability to zero of the state tracking error. We illustrate the robust performance of the designed GPID controllers by means of computer simulations including large unmodelled load parameter variations.

2 An Average model of a Boost Converter

Consider the boost converter circuit, shown in Figure 1. The system is described by the set of equations

$$\begin{aligned} L\dot{I} &= -uv + E \\ C\dot{v} &= uI - \frac{v}{R} \end{aligned} \tag{1}$$

where I represents the inductor current and v is the output capacitor voltage. The control input u, representing the switch position function, is a discrete-valued signal taking values in the set $\{0, 1\}$. The system parameters are constituted by: L, which is the inductance of the input circuit; C the capacitance of the output filter and R, the output load resistance. The external voltage source has the constant value E.

We introduce the following state normalization and time scale transformation:

$$x_1 = \frac{I}{E}\sqrt{\frac{L}{C}}, \quad x_2 = \frac{v}{E}, \quad \tau = \frac{t}{\sqrt{LC}} \tag{2}$$

The normalized model is thus given by:

$$\begin{aligned} \dot{x}_1 &= -ux_2 + 1 \\ \dot{x}_2 &= ux_1 - \frac{x_2}{Q} \\ y &= x_2 \end{aligned} \tag{3}$$

where now, with an abuse of notation, the " $\cdot$ " represents derivation with respect to the normalized time, τ. The variable x_1 is the normalized inductor current, x_2 is the normalized output voltage and u, still represents the switch position function. The constant system parameters are all comprised now in the circuit "quality" parameter, denoted by Q and given by the strictly positive quantity, $R\sqrt{C/L}$. It is assumed that the only system variable available for measurement is the output capacitor voltage x_2.

The *average state model* of the boost converter circuit, extensively used in the literature, may be directly obtained from the model (3) by simply identifying the switch position function u with the *duty ratio* function, denoted by μ, which is now a function restricted to take values in the closed interval $[0, 1]$. The average normalized inductor current and capacitor voltage are denoted, respectively by z_1 and z_2. We thus deal, from now on, with the following average normalized system equations which admit a physical interpretation, in terms of controlled voltage and current sources:

$$\begin{aligned}\dot{z}_1 &= -\mu z_2 + 1\\ \dot{z}_2 &= \mu z_1 - \frac{z_2}{Q}\\ y &= z_2\end{aligned} \tag{4}$$

2.1 Properties of the average normalized model

The average system (4) is differentially flat, with flat output given by the total normalized stored energy

$$F = \frac{1}{2}\left[z_1^2 + z_2^2\right] \tag{5}$$

Indeed, all system variables can be parameterized, modulo physical considerations, in terms of the flat output F and its first order time derivative $\dot{F}$.

$$\begin{aligned}z_1 &= -\frac{Q}{2} + \frac{1}{2}\sqrt{Q^2 + 4(Q\dot{F} + 2F)}\\ z_2 &= \sqrt{-Q\dot{F} - \frac{Q^2}{2} + \frac{Q}{2}\sqrt{Q^2 + 4(Q\dot{F} + 2F)}}\\ \mu &= \frac{1}{z_2\left(1 + \frac{2}{Q}z_1\right)}\left[-\frac{2}{Q}\dot{F} + \frac{1}{Q}\sqrt{Q^2 + 4(Q\dot{F} + 2F)} - \ddot{F}\right]\end{aligned} \tag{6}$$

An important property of the average model concerns the nature of the *zero dynamics* associated with the individual normalized average state variables. The variable z_2 is a non-minimum phase output while the variable z_1 is a minimum phase output (See Sira-Ramírez and Lischinsky-Arenas [9]). For this reason, in order to avoid internal instability problems, the feedback regulation of the average voltage, z_2, is usually carried out in an *indirect* fashion in terms of a corresponding regulation of z_1. It is also clear, from the strictly positive character of the flat output, F, and the assumption that, pointwise,

$\dot{F}(t) > -(2/Q)F(t)$, the average state and the average input variables are all strictly positive signals.

Henceforth, we concentrate in solving the stabilization and trajectory tracking problems for the described average normalized model of the Boost converter circuit. It is implied that a feedback solution of the average problem can be readily implemented, modulo some well-known approximation errors, on the actual switched system (1) by means of a suitable high frequency Pulse-Width-Modulated (PWM) feedback control scheme (see [8] and [10]).

3 GPID Regulation around a Nominal Trajectory

Suppose it is desired to achieve an equilibrium to equilibrium transfer for the non-minimum phase variable z_2, within a given finite interval of time $[t_0, t_1]$.

This problem is suitably transformed into a problem of adequately controlling the total stored energy F between the two corresponding equilibrium values. For this, note that if it is required to transfer the normalized capacitor voltage between the constant equilibrium values, $\overline{z}_2(t_0)$ and $\overline{z}_2(t_1)$, the corresponding equilibrium values for the flat output, $\overline{F}(t_0)$, $\overline{F}(t_1)$, are given, according to (6), by

$$\begin{aligned}
\overline{F}(t_0) &= \frac{1}{2}\left(\frac{\overline{z}_2^2(t_0)}{Q}\right)^2 + \frac{1}{2}\overline{z}_2^2(t_0) \\
\overline{F}(t_1) &= \frac{1}{2}\left(\frac{\overline{z}_2^2(t_1)}{Q}\right)^2 + \frac{1}{2}\overline{z}_2^2(t_1)
\end{aligned} \tag{7}$$

Thus, a nominal flat output trajectory $F^*(t)$ can be prescribed which smoothly and monotonically interpolates between the equilibrium values, $\overline{F}(t_0)$, and $\overline{F}(t_1)$. Note that this off-line planned prescription of the flat output, immediately renders, via use of (6) the nominal (open loop) normalized state and control input trajectories, $z_1^*(t)$, $z_2^*(t)$, $\mu*(t)$, without integrating any differential equations.

3.1 A time-varying linearized model around the nominal stabilizing trajectory

The jacobian linearization around the prescribed nominal stabilizing trajectory, characterized by the functions $z_1^*(t)$, $z_2^*(t)$ and $\mu^*(t)$, is readily obtained to be

$$\begin{aligned}
\dot{z}_{1\delta} &= -\mu^*(t)z_{2\delta} - z_2^*(t)\mu_\delta \\
\dot{z}_{2\delta} &= \mu^*(t)z_{1\delta} + z_1^*(t)\mu_\delta - \frac{1}{Q}z_{2\delta} \\
y_\delta &= z_{2\delta}
\end{aligned} \tag{8}$$

This linear time-varying system is written in matrix form: $\dot{z}_\delta = A(t)z_\delta + b(t)\mu_\delta$, $y_\delta = c(t)z_\delta$, as

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} = \begin{bmatrix} 0 & -\mu^*(t) \\ \mu^*(t) & -\frac{1}{Q} \end{bmatrix}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} + \begin{bmatrix} -z_2^*(t) \\ z_1^*(t) \end{bmatrix}\mu_\delta \tag{9}$$

The linearized system is uniformly controllable for all physically feasible trajectories ($z_1^*(t) > 0, z_2^*(t) > 0$). The controllability matrix is computed, according to the formula developed by Silverman and Meadows [7],

$$C(t) = [\ b(t), (A(t) - \frac{d}{dt})b(t)\ ] = \begin{bmatrix} -z_2^*(t) & -\frac{1}{Q}z_2^*(t) \\ z_1^*(t) & -1-\frac{1}{Q}z_1^*(t) \end{bmatrix} \tag{10}$$

The system loses controllability around the conditions: $z_2^*(t) = 0$ and $z_1^*(t) = -\frac{Q}{2}$, which are not physically significant. The system is thus uniformly controllable in the region of the state space of interest. Hence, the system is differentially flat, with flat output given by a time-varying linear combination of the state variables. In this case, such a flat output is given by:

$$F_\delta = z_1^*(t)z_{1\delta} + z_2^*(t)z_{2\delta} \tag{11}$$

which represents the incremental normalized energy around its nominal value: $F^*(t) = (1/2)([\,z_1^*(t)\,]^2 + [\,z_2^*(t)\,]^2)$. The flat output time derivative represents the incremental consumed power and it is given by

$$\dot{F}_\delta = z_{1\delta} - \frac{2z_2^*(t)}{Q}z_{2\delta} \tag{12}$$

Indeed, all the incremental system variables are differentially parameterizable in terms of F_δ and its time derivatives,

$$\begin{aligned}
z_{1\delta} &= \frac{1}{\left(1+\frac{2}{Q}z_1^*(t)\right)}\left[\frac{2}{Q}F_\delta + \dot{F}_\delta\right] \\
z_{2\delta} &= \frac{1}{z_2^*(t)\left(1+\frac{2}{Q}z_1^*(t)\right)}\left[F_\delta - z_1^*(t)\dot{F}_\delta\right] \\
\mu_\delta &= -\frac{1}{z_2^*(t)\left(1+\frac{2}{Q}z_1^*(t)\right)}\Bigg\{\frac{2}{Q}z_2^*(t)\mu^*(t)z_{1\delta} \\
&\quad + \left[\mu^*(t)\left(1+\frac{2}{Q}z_1^*(t)\right) - \frac{4}{Q^2}z_2^*(t)\right]z_{2\delta} + \ddot{F}_\delta\Bigg\} \\
&= \frac{1}{z_2^*(t)\left(1+\frac{2}{Q}z_1^*(t)\right)^2}\Bigg\{\left[\frac{4}{Q^2}z_2^*\mu^* + \frac{1}{z_2^*}\left(\mu^*(1+\frac{2}{Q}z_1^*) - \frac{4}{Q^2}z_2^*\right)\right]F_\delta \\
&\quad + \left[\frac{2}{Q}z_2^*\mu^* - \left(\mu*\left(1+\frac{2}{Q}z_1^*\right) - \frac{4}{Q^2}z_2^*\right)\frac{z_1^*}{z_2^*}\right]\dot{F}_\delta + \ddot{F}_\delta\Bigg\}
\end{aligned} \tag{13}$$

The linearized system, (9), is also uniformly observable from the output y_δ since, according to the Silverman-Meadows test [7], we obtain the following observability matrix:

$$\mathcal{O}(t) = \left[c^T \; ; \; (A - \frac{d}{dt})^T c^T\right] = \begin{bmatrix} 0 & \mu^*(t) \\ 1 & -\frac{1}{Q} \end{bmatrix} \tag{14}$$

and the system is seen to loose observability whenever the nominal duty ratio function satisfies $\mu^*(t) = 0$. It is clear, then, that the linearized system (9) is constructible and the unmeasured state can be expressed in terms of integrals of linear time-varying combinations of the incremental input μ_δ and the incremental output y_δ. Indeed, from the linearized system equations (8) we have,

$$\begin{aligned} \widehat{z}_{1\delta}(t) &= -\int_0^t \left[\, \mu^*(\sigma) y_\delta(\sigma) + z_2^*(\sigma)\mu_\delta(\sigma)\right] d\sigma \\ z_{2\delta}(t) &= y_\delta(t) \end{aligned} \tag{15}$$

The relation linking the structural estimate of the incremental normalized inductor current, $\widehat{z}_{1\delta}$, with its actual value, $z_{1\delta}$, is given by

$$z_{1\delta} = \widehat{z}_{1\delta} + z_{1\delta}(0) \tag{16}$$

Similarly, the relations between the structural estimates of the incremental flat output and of its first order time derivative, and their actual values are given, according to (11), (12), by,

$$\begin{aligned} F_\delta &= \widehat{F}_\delta + z_1^*(t) z_{1\delta}(0) \\ \dot{F}_\delta &= \widehat{\dot{F}}_\delta + z_{1\delta}(0) \end{aligned} \tag{17}$$

3.2 A Lyapunov based controller

Consider the time-invariant Lyapunov function candidate, $V(z_{1\delta}, z_{2\delta})$ given by.

$$V(z_{1\delta}, z_{2\delta}) = \frac{1}{2}\left[z_{1\delta}^2 + z_{2\delta}^2\right]$$

The time derivative of the Lyapunov function, along the trajectories of the linearized system yield

$$\dot{V}(z_{1\delta}, z_{2\delta}, \mu, t) = -\frac{z_{2\delta}^2}{Q} + \left(z_1^*(t) z_{2\delta} - z_2^*(t) z_{1\delta}\right) \mu_\delta \tag{18}$$

The time-varying controller

$$\mu_\delta = -\gamma \left(z_1^*(t) z_{2\delta} - z_2^*(t) z_{1\delta}\right) \tag{19}$$

with $\gamma > 0$, results in a negative definite time derivative of $V(z_{1\delta}, z_{2\delta})$ along the closed-loop controlled trajectories of the system,

$$\dot{V}(z_{1\delta}, z_{2\delta}, t) = -\frac{z_{2\delta}^2}{Q} - \gamma\left(z_1^*(t)z_{2\delta} - z_2^*(t)z_{1\delta}\right)^2 \tag{20}$$

The closed loop system (8), (19) is then an exponentially asymptotically stable linear time-varying system for any strictly positive design parameter γ.

Use of the structural estimate for $z_{1\delta}$, given by (15), in the Lyapunov based controller, (19) leads, after appropriate integral output error compensation, to the equivalent closed-loop incremental system:

$$\begin{aligned}
\dot{z}_{1\delta} &= -\gamma[z_2^*(t)]^2 z_{1\delta} - [\mu^*(t) - \gamma z_1^*(t)z_2^*(t)]z_{2\delta} + \gamma[z_2^*(t)]^2(z_{10\delta} - k\zeta_\delta) \\
\dot{z}_{2\delta} &= [\mu^*(t) + \gamma z_1^*(t)z_2^*(t)]z_{1\delta} - (\frac{1}{Q} + \gamma[z_1^*(t)]^2)z_{2\delta} \\
&\quad + \gamma z_1^*(t)z_2^*(t)(z_{10\delta} - k\zeta_\delta) \\
\dot{\zeta}_\delta &= z_{2\delta}
\end{aligned} \tag{21}$$

Letting $\rho_\delta = (z_{10\delta} - k\zeta_\delta)$, we obtain the following matrix representation of the closed loop system:

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ \rho_\delta \end{bmatrix} = \begin{bmatrix} -\gamma[z_2^*(t)]^2 & -[\mu^*(t) - \gamma z_1^*(t)z_2^*(t)] & \gamma[z_2^*(t)]^2 \\ [\mu^*(t) + \gamma z_1^*(t)z_2^*(t)] & -(\frac{1}{Q} + \gamma[z_1^*(t)]^2) & \gamma z_1^*(t)z_2^*(t) \\ 0 & -k & 0 \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ \rho_\delta \end{bmatrix} \tag{22}$$

The point-wise eigenvalues of the closed loop system, (22), are guaranteed to be bounded away from the imaginary axis, in the open left portion of the complex plane, provided the constant gain k is chosen to satisfy the rather conservative, but feasible, condition:

$$0 < k < \frac{1}{Q\gamma z_{2\,\max}^*(t)}$$

It is relatively straightforward then to show that the closed loop system, (22), satisfies the hypothesis in the following Theorem due to Rugh [6]. This, as usual, entitles a sufficiently slow and sufficiently differentiable nominal equilibrium to equilibrium transfer trajectory for the flat output. The closed loop system, (22), is then seen to be exponentially asymptotically stable.

Theorem 1. (Rugh [pp. 135-138]).

Suppose that for the linear time-varying system $\dot{x} = A(t)x$, the matrix $A(t)$ is continuously differentiable and there exist finite positive constants α and δ, such that, for all t, $\|A(t)\| \leq \alpha$, and every pointwise eigenvalue of $A(t)$ satisfies $Re[\lambda(t)] \leq -\delta$. Then, there exist a positive constant β such that if the time derivative of $A(t)$ satisfies $\|\dot{A}(t)\| \leq \beta$, for all t, the state equation is uniformly exponentially stable.

4 Conclusions and Suggestions for Further Research

In this article, we have presented a GPID control scheme for stabilizing trajectory tracking tasks in an average nonlinear model of a DC-to-DC power converter of the "Boost" type. Eventhough the presented developments are cast in the context of linearized time-varying average models, the ideas can be extended to the full nonlinear case. The flexibility of the GPID controller approach to accommodate to any linear state feedback controller design technique, has been illustrated by using Lyapunov-based controllers. The performance of the GPID feedback controlled systems was evaluated by means of digital computer simulations with highly satisfactory results.

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Figures

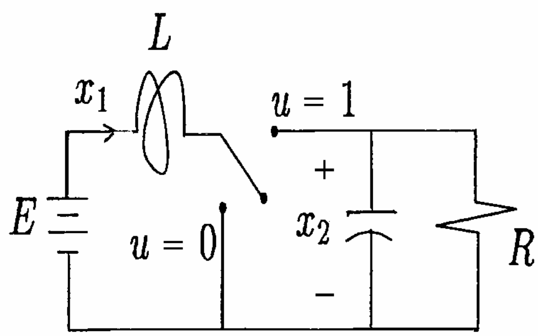

Fig. 1. The boost converter circuit

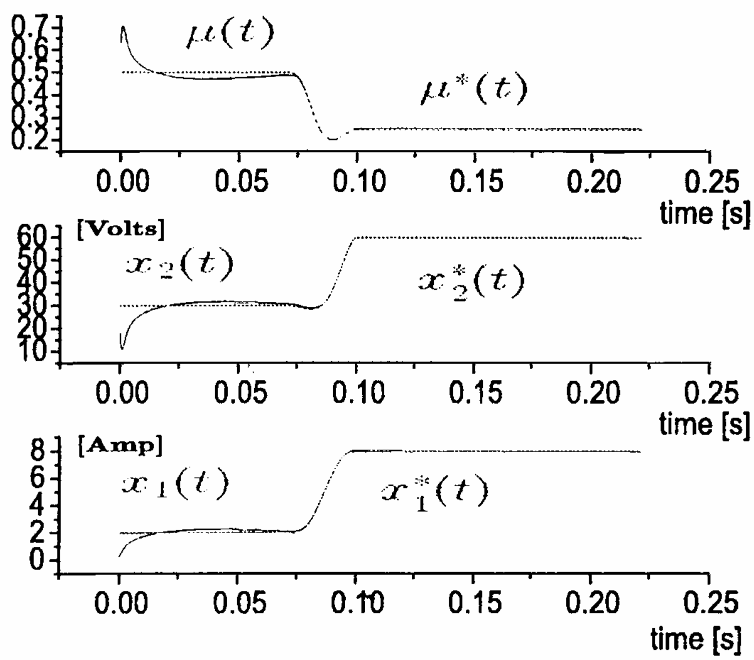

Fig. 2. Generalized PID controlled responses around an equilibrium to equilibrium transfer trajectory

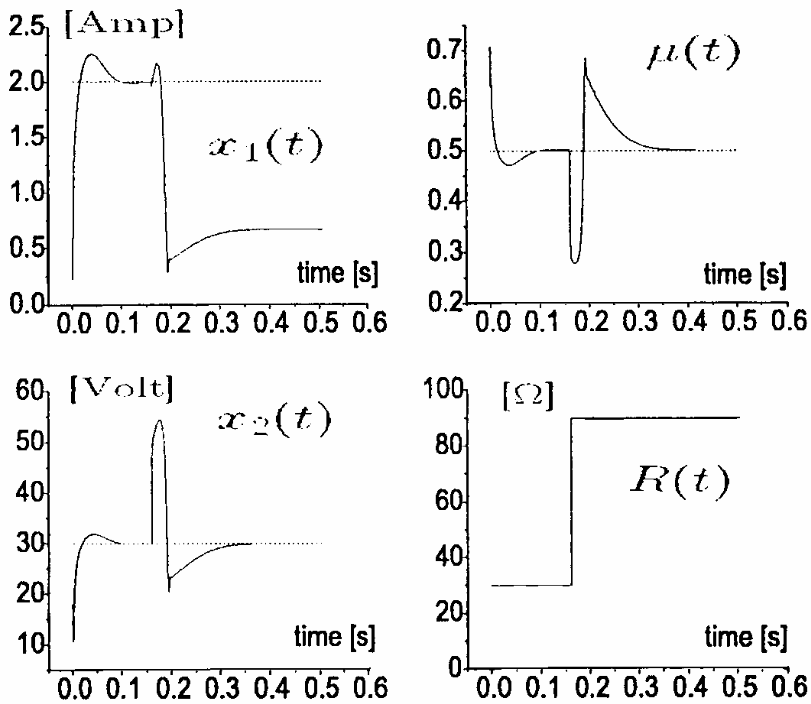

Fig. 3. Generalized time-varying PID controlled responses to an unmodelled, and permanent, load change