On the Generalized PI control of some nonlinear mechanical systems [1]

Hebertt Sira-Ramírez
CINVESTAV-IPN
Avenida IPN, # 2508, Col. San Pedro Zacatenco, A.P. 14740
México, D.F., México.
email hsira@mail.cinvestav.mx

Abstract

In this article, we present a generalized proportional-integral (GPI) control approach for the regulation, and trajectory tracking, problems defined on several nonlinear mechanical systems of the manipulator type. In all cases, the feedback control law can be realized by measuring only a link position. No asymptotic observers, nor time discretizations, are therefore needed in the feedback loop for the estimation of angular, or translational, velocities commonly required in the traditional state-based feedback controllers for such systems.

Keywords: Generalized PID control, mechanical manipulator systems.

1 Introduction

One of the main drawbacks of modern control theory is constituted by the need to completely measure the state of the system, or to estimate it by means of asymptotic observers. In practice, one frequently resorts to calculations based on high frequency samplings of the measured output signals. Either approach reduces the effectiveness of the preferred feedback control scheme. For the continuous regulation of linear systems, the need for state observers, or time discretizations, has been recently elegantly side-stepped (see Fliess *et al*, [2], Marquez *et al* [4]) by the introduction of a new technique called "Generalized PI Control" (GPI). The theoretical developments of this novel theory have been initially restricted to linear time-invariant systems. The advent of a nonlinear theory of GPI control has been recently announced in Fliess [1].

[1]This research was supported by the Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, (CINVESTAV-IPN), of Mexico and by the Consejo Nacional de Investigaciones Científicas y Tecnológicas, (CONACYT), under Research Contract, No. 32681-A.

In this article, we extend the GPI control technique for the regulation of some nonlinear mechanical manipulators. We show that, in the treated cases, the GPI control option can be directly used in a rather systematic fashion. The class of nonlinear systems that can directly benefit from this control technique seems to be the class of nonlinear systems admitting nonlinear asymptotic observers with exact linear state reconstruction errors achieved by nonlinear output injections. However, the GPI technique is superior to the observer based controller with regard to non-modeled perturbation inputs and sudden system parameter variations as already demonstrated in Fliess *et al*[3].

2 Controlling a single link manipulator

Consider the single link manipulator, shown in Figure 1, described by the second order controlled differential equation:

$$ml^2\ddot{\theta} - mgl\sin\theta = T \tag{2.1}$$

where m is the mass of the link, l is the length of the link and T is the applied torque. The constant gravity acceleration is denoted by g.

We first *normalize* the system equations in order to simplify the presentation. For this, we take a new time scale τ and a normalized control input u, given by

$$\tau = \left(\sqrt{g/l}\right) t, \qquad u = T/mgl \tag{2.2}$$

The normalized, dimensionless, model is thus given by

$$\ddot{\theta} - \sin\theta = u \tag{2.3}$$

where, with some abuse of notation, the " dot " now stands for time derivation with respect to the normalized time scale τ.

A direct exact linearization based feedback controller design, for accomplishing the tracking of a given desired angular trajectory: $\theta^*(\tau)$, is given by

$$u = -\sin\theta + \ddot{\theta}^*(\tau) - k_2(\dot{\theta} - \dot{\theta}^*(\tau)) - k_1(\theta - \theta^*(\tau)) \quad (2.4)$$

Upon appropriate choice of the design parameters k_2 and k_1, the preceding controller imposes the following exponentially asymptotically stable closed loop dynamics on the tracking error $e = \theta - \theta^*(\tau)$

$$\ddot{e}(\tau) + k_2\dot{e}(\tau) + k_1 e(\tau) = 0 \quad (2.5)$$

The tracking controller (2.4) requires the knowledge of the angular velocity variable $\dot{\theta}$. This means that such a variable has to be either measured, or estimated by means of an observer. In practise, the estimation requires the use of on-line calculations based on high frequency samples of the position variable trajectory $\theta(\tau)$. Using generalized PID control, such estimations, or time discretizations, are unnecessary.

Indeed, suppose that only the link position $\theta(\tau)$ is available for measurement. Observe, then, that the angular velocity, $\dot{\theta}(\tau)$, can be directly computed from (2.3) as

$$\dot{\theta}(\tau) = \int_0^\tau \sin(\theta(\rho))d\rho + \int_0^\tau u(\rho)d\rho + \dot{\theta}(0) \quad (2.6)$$

where $\dot{\theta}(0)$ is the value of the angular velocity, at time $\tau = 0$. Such information, however, is usually not available either and we can only assume that this quantity is known in the simplest of cases (i.e. when it is known that the link starts with zero velocity). Therefore, we assume no knowledge of $\dot{\theta}(0)$.

Suppose, for a moment, that motivated by the simplicity of (2.6), we insist upon using the following "faulty" estimate, or "structural estimate" of the angular velocity, $\dot{\theta}(\tau)$,

$$\widehat{\dot{\theta}}(\tau) = \int_0^\tau [\sin(\theta(\rho)) + u(\rho)]\, d\rho \quad (2.7)$$

Certainly, the estimate (2.7) has the interesting advantage of being easily synthesized using only the integral of a linear combination of the input and a nonlinear function of output. Evidently, the exact relationship linking the structural estimate $\widehat{\dot{\theta}}(\tau)$ to the actual value of the angular velocity, $\dot{\theta}(\tau)$, is given by

$$\dot{\theta}(\tau) = \widehat{\dot{\theta}}(\tau) + \dot{\theta}(0) \quad (2.8)$$

Therefore, if we use (2.7) in the controller expression (2.4), we obtain the following equivalent feedback law,

$$\begin{aligned} \hat{u} &= -\sin\theta + \ddot{\theta}^*(\tau) - k_2(\widehat{\dot{\theta}}(\tau) - \dot{\theta}^*(\tau)) - k_1(\theta - \theta^*(\tau)) \\ &= -\sin\theta + \ddot{\theta}^*(\tau) - k_2\left(\int_0^\tau [\sin(\theta(\rho)) + u(\rho)]\, d\rho\right. \\ &\quad \left. - \dot{\theta}^*(\tau)\right) - k_1(\theta - \theta^*(\tau)) \qquad (2.9) \end{aligned}$$

When the feedback control law (2.9) is used on the system (2.3), one obtains, after use of (2.8), the following closed loop trajectory tracking error dynamics

$$\ddot{e}(\tau) + k_2\dot{e}(\tau) + k_1 e(\tau) = k_2\dot{\theta}(0) \quad (2.10)$$

which exhibits an "off-set" tracking error due to a constant initial condition excitation of the asymptotically stable tracking error dynamics. This immediately prompts us to consider the possibility of using a modified nonlinear feedback controller including an *integral error* feedback control term. We thus proceed to propose the following controller:

$$\begin{aligned} u &= -\sin\theta + \ddot{\theta}^*(\tau) - k_1(\theta - \theta^*(\tau)) + \xi \\ &\quad - k_2\left(\int_0^\tau [\sin(\theta(\rho)) + u(\rho)]\, d\rho - \dot{\theta}^*(\tau)\right) \\ \dot{\xi} &= -k_0(\theta - \theta^*(\tau)) \qquad (2.11) \end{aligned}$$

Use of the modified controller (2.11) on (2.3) results, after use of (2.8), in the following closed loop tracking error system:

$$\ddot{e} + k_2\dot{e} + k_1 e = k_2\dot{\theta}(0) - k_0\int_0^\tau e(\rho)d\rho \quad (2.12)$$

which is, evidently, equivalent upon differentiation to the third order linear tracking error dynamics,

$$e^{(3)} + k_2\ddot{e} + k_1\dot{e} + k_0 e = 0 \quad (2.13)$$

No doubt, the obtained tracking error dynamics, (2.13), can be rendered exponentially asymptotically stable by appropriate choice of the design constants $\{k_2, k_1, k_0\}$.

We summarize the previous development in the following proposition.

Proposition 2.1 *Given a desired angular displacement trajectory $\theta^*(\tau)$ for the nonlinear single link manipulator system (2.3), then the GPI controller (2.11) globally exponentially asymptotically stabilizes the tracking error $e = \theta - \theta^*(\tau)$ to zero, provided the design gains $\{k_2, k_1, k_0\}$ are chosen so that the polynomial $p(s)$ in the complex variable s,*

$$p(s) = s^3 + k_2 s^2 + k_1 s + k_0$$

is Hurwitz.

2.1 Simulation results

Figure 2 shows a block diagram of the GPI feedback control scheme (2.11) for the trajectory tracking task. Figure 3 depicts the performance of the proposed feedback controller for a manipulator characterized by the parameters $m = 0.5$ [Kg], $l = 0.4$ [m], $g = 9.8$ [m/w^2]. The control task consisted in maneuvering the link from the initial position $\theta(t_1) = \pi$ [rad] towards the final position $\theta(t_2) = -\pi/2$ [rad], in a (non-normalized) time interval of $t_2 - t_1 = 0.8$[s], with $t_1 = 0.4$ [s] and $t_2 = 1.2$ [s]. The controller parameters were chosen so that the closed loop characteristic polynomial for the normalized system coincided with $(s^2 + 2\chi\omega_n s + \omega_n^2)(s + \beta)$, with $\chi = 0.85$, $\omega_n = 2$, $\beta = 2$. The nominal trajectory for the link position $\theta^*(\tau)$ was specified by means of a Bézier polynomial, smoothly interpolating between the initial and final values.

$$\begin{aligned}\theta^*(\tau) &= \theta(\tau_1) + (\theta(\tau_2) + \theta(\tau_1)) \left[\frac{\tau - \tau_1}{\tau_2 - \tau_1}\right]^5 \times \\ &\left[r_1 - r_2\left(\frac{\tau - \tau_1}{\tau_2 - \tau_1}\right) + \cdots - r_6\left(\frac{\tau - \tau_1}{\tau_2 - \tau_1}\right)^5\right]\end{aligned} \tag{2.14}$$

with

$$r_1 = 252, \quad r_2 = 1050, \quad r_3 = 1800, \quad r_4 = 1575,$$

$$r_5 = 700, \quad r_6 = 126$$

3 The gyroscopic pendulum

Consider the gyroscopic pendulum, shown in Figure 4, which has been extensively treated in the recent literature about the control of nonlinear mechanical systems (See Spong *et al*, [7] and Spong *et al* [5]).

A normalized model of such system reads

$$\begin{aligned}\ddot{\theta}_1 &= \frac{1}{1-\epsilon}[\sin\theta_1 - \epsilon u] \\ \ddot{\theta}_2 &= \frac{1}{1-\epsilon}[-\sin\theta_1 + u]\end{aligned} \tag{3.1}$$

where ϵ is a dimensionless quantity and the " $\cdot$ ", stands for derivation with respect to the normalized time.

3.1 Some analysis based on the flatness of the system

The normalized gyroscopic pendulum system is differentially flat, with flat output given by the quantity:

$$F = \theta_1 + \epsilon\theta_2 \tag{3.2}$$

This artificial output has the interpretation of the Huygen's center of oscillation of the mechanical system and it completely parameterizes all system variables, including the input. Indeed, the system variables can be written as *differential functions* of F as follows:

$$\begin{aligned}\theta_1 &= \arcsin\left(\ddot{F}\right), \quad \dot{\theta}_1 = \frac{F^{(3)}}{\sqrt{1-(\ddot{F})^2}} \\ \theta_2 &= \frac{1}{\epsilon}\left[F - \arcsin\left(\ddot{F}\right)\right], \\ \dot{\theta}_2 &= \frac{1}{\epsilon}\left[\dot{F} - \frac{F^{(3)}}{\sqrt{1-(\ddot{F})^2}}\right]\end{aligned}$$

$$u = \frac{1}{\epsilon}\left[\ddot{F} - (1-\epsilon)\left(\frac{F^{(4)}\left(1-(\ddot{F})^2\right) + \ddot{F}(F^{(3)})^2}{(1-(\ddot{F})^2)^{3/2}}\right)\right] \tag{3.3}$$

The differential parameterization (3.3) allows us to establish some general properties of the system. From the first equation in (3.3) it is clear that the pendulum position θ_1 is a non-minimum phase output. Indeed, a constant value of the pendulum angular position $\theta_1 = \overline{\theta}_1$ induces a corresponding flat output trajectory which satisfies the unstable dynamics: $\ddot{F} = \sin(\overline{\theta}_1)$. This means that $\dot{F}$ increases linearly. Since θ_1 is fixed, this, in turn, implies that the disk position θ_2 is increasing without limit. The second equation in (3.3) for $\theta_1 = \overline{\theta}_1$ implies that $F^{(3)}(\tau) = 0$. Substituting this value in the expression for $\dot{\theta}_2$ we obtain that the angular velocity of θ_2 is proportional to the flat output time derivative, i.e. $\dot{\theta}_2 = (1/\epsilon)\dot{F}$, and, hence $\theta_2 = (1/\epsilon)[F - \overline{\theta}_1]$. The nominal value of the control input u becomes proportional to the flat output acceleration, $u = (1/\epsilon)\ddot{F}$. The particular value of interest for $\overline{\theta}_1$ is the one corresponding to the "straight up" position, i.e. $\overline{\theta}_1 = 0$. This means that the angular velocity, $\overline{\theta}_2$, will become constant and the control input will be nominally zero at this position. The unstable behavior of θ_2 surely causes no problem since it only means that the disk will be turning at a constant speed.

3.2 Control objective and off-line trajectory planning

A control task consists in swinging up the pendulum from its stable downwards position towards its unstable straight up position. At time $\tau = \tau_0$, the pendulum is at rest at the downwards stable equilibrium position. The "swing up" maneuver is to be accomplished during the interval $[\tau_1, \tau_2]$, with $\tau_1 > \tau_0$. The desired angular displacement requires that the angular position starts at the value $\theta_1(\tau_1) = \pi$ [rad], with zero angular velocity, $\dot{\theta}_1(\tau_1) = 0$ [rad/tu] (*tu* stands for normalized "time units"), and it is required to end, after a finite period of time T, at the value $\theta_1(\tau_1 + T) = \theta_1(\tau_2) = 0$ [rad], with zero angular velocity, $\dot{\theta}_1(\tau_1 + T) = \dot{\theta}_1(\tau_2) = 0$ [rad/tu]. The nominal desired maneuver can therefore be specified by a time polynomial function, $\varphi(\tau, \tau_1, \tau_2)$ of the Bézier type which "smoothly" interpolates between the initial and final values for the nominal pendulum link angular displacement, here denoted by $\theta_1^*(\tau)$. The choice of the form of the polynomial spline is rather arbitrary. We specify it as follows

$$\theta_1^*(\tau) = \pi\left[1 - \varphi(\tau, \tau_1, \tau_2)\right] \tag{3.4}$$

with $\varphi(\tau, \tau_1, \tau_2)$ being a time-polynomial given, for instance, by the Bézier polynomial (2.14) used in Section 2.1.

The disk displacement nominal trajectory, corresponding with the desired angular displacement $\theta_1^*(\tau)$, can be computed using the flatness property. This task is more directly accomplished in this manner than by using the system differential equations. The flat output nominal trajectory is obtained by direct integration of the following differential equation

$$\ddot{F}^*(\tau) = \sin(q_1^*(\tau)) \tag{3.5}$$

The initial conditions for the flat output and its first time derivative are readily obtained from the definition of the flat output and the nominal initial conditions for $q_1^*(\tau)$ as, $\dot{F}^*(\tau_1) = \epsilon\dot{q}_2^*(\tau_1)$ and $F^*(\tau_1) = q_1^*(\tau_1) + \epsilon q_2^*(\tau_1)$. We let the disc be initially at rest, prior to time τ_1, with $\dot{q}_2^*(\tau_1) = 0$ and we let $q_2^*(\tau_1)$ to be such that $F^*(\tau_1)$ is zero, Summarizing we have

$$\dot{F}^*(\tau_1) = 0, \quad F^*(\tau_1) = 0, \quad q_2^*(\tau_1) = -\frac{1}{\epsilon}q_1^*(\tau_1) = -\frac{\pi}{\epsilon}$$

The nominal trajectory for the flat output, $F^*(\tau)$, is then obtained as:

$$F^*(\tau) = \begin{cases} 0 & \text{for all } \ \tau \leq \tau_1 \\ \int_{\tau_1}^{\tau}\int_{\tau_1}^{\rho}\sin(\theta^*(\lambda))d\lambda d\rho & \text{for all } \ \tau \geq \tau_1 \end{cases} \tag{3.6}$$

The nominal trajectory for the variable q_2 is determined by the relation,

$$q_2^*(\tau) = \frac{1}{\epsilon}\left[F^*(\tau) - \theta_1^*(\tau)\right] \tag{3.7}$$

3.3 Feedback controller design

Given the inescapable, and possibly harmless, non-minimum phase nature of the link position variable, θ_1, we immediately realize that we can proceed to directly control θ_1, without resorting to the flatness property, other than in establishing the nominal trajectories and the performed analysis. (Incidentally the flatness based controller exhibits a singularity at any multiple of the value $\theta_1 = \pi/2$). We make no specific regard for the evolution of θ_2, other than the reasonable requirement of having a final *constant* angular speed, $\dot{\theta}_2$, for the rotating disk. This, as already established from the flatness property, is guaranteed only at the upright and the downwards equilibrium positions.

We consider the following exactly linearizing feedback controller,

$$\begin{aligned} u \;=\; & \frac{1}{\epsilon}\sin\theta_1 \\ & -\frac{1-\epsilon}{\epsilon}\left[\ddot{\theta}_1^*(\tau) - k_2(\dot{\theta}_1 - \dot{\theta}_1^*(\tau)) - k_1(\theta_1 - \theta_1^*(\tau))\right] \end{aligned} \tag{3.8}$$

This controller produces the closed loop tracking error dynamics

$$\ddot{e} + k_2\dot{e} + k_1 e = 0 \tag{3.9}$$

with $e = \theta - \theta^*(\tau)$ being the angular position tracking error.

In spite of its simplicity, the controller (3.8) requires the knowledge of the angular velocity $\dot{\theta}_1$, which we have assumed to be unavailable. We proceed to find a suitable integral input-output parameterization of the angular velocity variable.

3.4 Integral input-output parameterization and GPI controller

We obtain an integral input-output parameterization of the pendulum's angular velocity $\dot{\theta}_1(\tau)$, which avoids the need for using observers, or on line calculations based on time discretizations of the pendulum position variable trajectory.

Integrating once the first equation in the normalized system (3.1), we obtain:

$$\dot{\theta}_1(\tau) = \frac{1}{1-\epsilon}\left[\int_0^{\tau}\sin(\rho)d\rho - \epsilon\int_0^{\tau}u(\rho)d\rho\right] \tag{3.10}$$

We denote such a structural estimate of $\dot{\theta}_1$ by $\widehat{\dot{\theta}_1}$, and rewrite it in its simpler form as,

$$\widehat{\dot{\theta}_1}(\tau) = \frac{1}{1-\epsilon}\left[\int_0^{\tau}\left(\sin(\rho) - \epsilon u(\rho)\right)d\rho\right] \tag{3.11}$$

The exact relation of the angular velocity structural estimate (3.11) with the actual link angular velocity, $\dot{\theta}_1$, is given by

$$\widehat{\dot{\theta}_1}(\tau) = \dot{\theta}_1(\tau) - \dot{\theta}_1(0) \qquad (3.12)$$

where $\dot{\theta}_1(0)$ denotes the unknown initial angular velocity of the pendulum link.

Using the structural estimate of $\dot{\theta}_1$ in the controller (3.8) and complementing with integral control action based only on the output tracking error we obtain the following GPI controller:

$$\begin{aligned} u &= \frac{1}{\epsilon}\sin\theta_1 - \frac{1-\epsilon}{\epsilon}\left[\ddot{\theta}_1^*(\tau) - k_1(\theta_1 - \theta_1^*(\tau)) + \xi\right] \\ &\quad -\frac{1}{1-\epsilon}\left\{k_2\left(\left[\int_0^\tau (\sin(\theta_1(\rho)) - \epsilon u(\rho))\, d\rho\right] - \dot{\theta}_1^*(\tau)\right)\right\} \\ \dot{\xi} &= -k_0(\theta_1 - \theta_1^*(\tau)) \end{aligned} \qquad (3.13)$$

The closed loop tracking error dynamics is now given, after use of (3.12), by

$$\begin{aligned} \ddot{e} + k_2\dot{e} + k_1 e &= k_2\dot{\theta}_1(0) + \xi \\ \dot{\xi} &= -k_0 e \end{aligned} \qquad (3.14)$$

The characteristic polynomial of the closed loop system (3.14) is given by:

$$e^{(3)} + k_2\ddot{e} + k_1\dot{e} + k_0 e = 0 \qquad (3.15)$$

Evidently (3.15) can be made globally exponentially asymptotically stable by proper choice of the set of design parameters, $\{k_2, k_1, k_0\}$.

We summarize the previous development in the following proposition.

Proposition 3.1 *Suppose a desired angular displacement trajectory $\theta_1^*(\tau)$ is given, for the nonlinear normalized gyroscopic system (3.1), which takes the pendulum link from its resting downward position towards the unstable upright position. Then, the GPI feedback controller, (3.13), globally exponentially asymptotically stabilizes the tracking error $e = \theta_1 - \theta_1^*(\tau)$ to zero, provided the design gains $\{k_2, k_1, k_0\}$ are chosen so that the polynomial $p(s)$ in the complex variable s,*

$$p(s) = s^3 + k_2 s^2 + k_1 s + k_0$$

is Hurwitz. The controlled motions exhibit an uniformly increasing disk position residual behavior, characterized by a constant *steady state inertial disk angular velocity.*

Figure 5 shows the performance of the controller for a manipulator characterized by the following parameters, taken from [[7]], $m_1 = 0.02$ [Kg], $m_2 = 0.063$ [Kg], $l_1 = 0.125$ [m], $l_{c1} = 0.063$ [m], $I_1 = 47 \times 10^{-6}$ [Kg-m^2], $I_2 = 32 \times 10^{-6}$ [Kg-m^2], $g = 9.8$ [m/s^2]. The control task consisted in maneuvering the pendulum link from the initial position, $\theta_1(\tau_1) = \pi$ [rad] towards the final upwards position, $\theta_1(\tau_2) = 0$ [rad], in a (non-normalized) time interval of $t_2 - t_1 = 1$[s], with $t_1 = 1$ [s] and $t_2 = 2$ [s]. The controller parameters were chosen so that the closed loop characteristic polynomial for the normalized system coincided with $(s^2 + 2\chi\omega_n s + \omega_n^2)(s + \beta)$, with $\chi = 0.85$, $\omega_n = 1$, $\beta = 2$. The nominal trajectory for the pendulum link anular gposition $\theta_1^*(\tau)$ was specified by means of the same Bézier polynomial used in the previous example.

4 Conclusions

In this article, we have extended the use of GPI controllers for the trajectory tracking of some mechanical robotic manipulators. The idea, originally developed within the context of linear systems (see [4]) has been shown here to be also applicable to a certain class of nonlinear mechanical systems. The developed feedback controllers are either based on exact linearization, exploiting the system flatness , or on partial linearization, as in the gyroscopic pendulum. The control synthesis only uses the available measurement of a single angular position output signal and the knowledge of the applied torque input. The scheme renders state observers and time discretizations completely unnecessary for the computation of the system's angular velocities. Such quantities are not required to be known in a precise fashion and only "structural estimates" are needed. The offset errors incurred in using such structural estimates in the controller are readily compensated in the feedback loop by adding suitable integral output tracking error feedback control actions.

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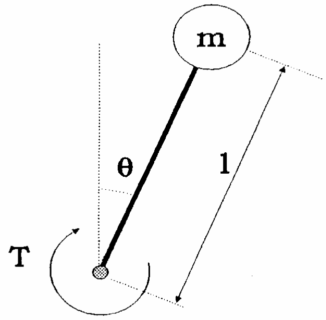

Figure 1: A single link manipulator.

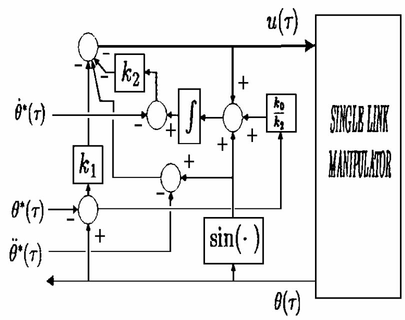

Figure 2: Control scheme for trajectory tracking in single link manipulator.

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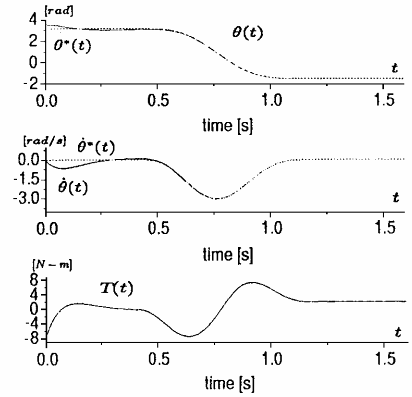

Figure 3: Performance of the Generalized PID controller.

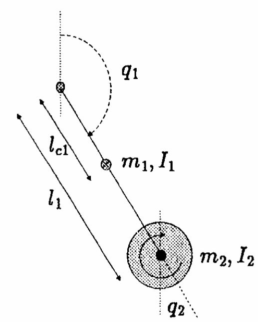

Figure 4: The gyrocopic pendulum.

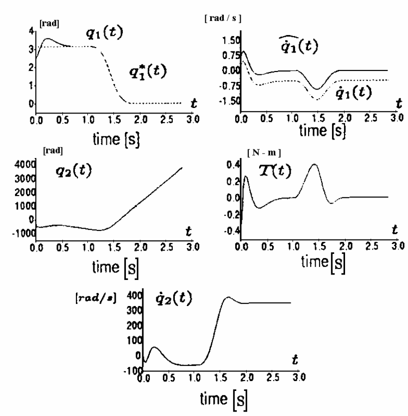

Figure 5: Performance of Generalized PID Feedback control on the gyroscopic pendulum.