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On the control of the resonant converter : A hybrid-flatness approach *

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Abstract

In this article we show that the series resonant DC/DC converter, which is a hybrid system, is piecewise differentially flat with a flat output which is invariant with respect to the structural changes undergone by the system evolution. This fact considerably simplifies the design of a switching output feedback controller that can be essentially solved by linear techniques. Flatness clearly explains all practical issues associated with the normal operation of the converter. Experimental results are presented which further evidence the actual applicability of the technique.

Keywords Resonant Converter, Hybrid Systems, Flat Systems.

1 Summary

In this article, we approach the regulation problem of a popular DC/DC power converter, known as the "series resonant converter", from the combined perspective of differential flatness and hybrid systems. The converter is a variable structure system with a linear controllable model in each one of the two *locations*, or regions, of the systems hybrid state space. On each constitutive location of the corresponding hybrid automaton, the system is thus represented by a *flat* system. The flat output expression of the system, in terms of the state variables, is distinctively marked by the hybrid character of the system. However, the differential relation existing between the flat output and the control input is *invariant* throughout the set of locations. By resorting to flatness, one clearly shows that the circuit variables which are required to achieve resonance (i.e. sinusoidal oscillatory behavior) also exhibit invariant differential parameterizations, in terms of the flat output. These two facts considerably simplify the hybrid controller design problem for both the "start up" phase and the steady state energy set point regulation phase of the converter. The regulation of the steady state oscillations entitle switching on a hyperplane whose synthesis requires knowledge of the resonant state variables. The practical limitations on the availability of such measurements is greatly alleviated by the fact that the controllable flat output dynamics is also observable from the only measurable

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output. This implies that the "start up" process, and the synthesis of a stable oscillatory behavior, can be entirely carried out by means of an output-based hybrid feedback controller using either an asymptotic observer or using an hybrid version of an integral state reconstructor, of recent introduction in the control systems literature (See Fliess et al [3]).

2 The series resonant DC/DC power converter

Resonant converters have been the object of sustained interest throughout the last two decades. Roughly speaking, the controller design for such hybrid systems has been approached from different viewpoints including: an approximate DC viewpoint, a phase plane approach, averaging methods defined on phasor variable methods and, more recently, from a passivity based approach.

Approximate analysis, based on DC considerations, was undertaken in Vorpérian and Cúk [13] [14]. These tools are rather limited given the hard nonlinear nature of the converter. Control strategies based on state variable representations were initiated in Oruganti and Lee in [15], [5]. These techniques were clearly explained later, on a simplified converter model, in Rossetto [6]. An optimal control approach was developed in Sendanyoye *et al* [10] and a similar approach was reported in the work of Oruganti *et al* [7]. Several authors have also resorted to either exact or approximate discretization strategies as in Verghese *et al* [12] and in Kim *et al* [4]. A phasor transformation approach was provided in the work of Rim and Cho [8], which is specially suited for DC to AC conversion. An interesting averaging method, based on local Fourier analysis, has been presented in an article by Sanders *et al* [9]. These frequency domain approximation techniques have also found widespread use in other areas of power electronics. Using this approach, approximate schemes relying on Lyapunov stability analysis and the passivity based control approach, have been reported, respectively, in the works of Stankovic *et al* [11] and Escobar [1].

Our approach is fundamentally based in the concept of *differential flatness* introduced few year ago in Fliess *et al* [2]. The flatness property, exhibited by many systems of practical interest, is here exploited to obtain, from its simple linear dynamics, suitable estimates, or integral reconstructors, of the converter state variables by means of linear design techniques.

Figure 1 shows a simplified nonlinear circuit representing the series resonant DC/DC power converter. The controlled nonlinear differential equations modeling the circuit are given by

$$\begin{aligned} L\frac{di}{dt} &= -v - v_0 \text{ sign } (i) + E(t) \\ C\frac{dv}{dt} &= i \\ C_0\frac{dv_0}{dt} &= \text{abs } (i) - \frac{v_0}{R} - I_0 \end{aligned} \tag{2.1}$$

where v and i are, respectively, the series capacitor voltage and the inductor current in the resonant series tank, while v_0 is the output capacitor voltage feeding both the load R and the sink current I_0 which, for simplicity, we assume to be of value zero. The input to the system is $E(t)$, which is usually restricted to take values in the discrete set $\{-E, E\}$ where E is a fixed given constant.

The objective is to attain a nearly constant voltage across the load resistance R on the basis of the rectified, and low-pass filtered, sinusoidal inductor current signal internally generated by the system in the L, C series circuit with the suitable aid of the amplitude restricted control input signal.

One readily obtains the following *normalized* model of the resonant circuit equations (2.1).

$$\begin{aligned} \dot{z}_1 &= -z_2 - z_3 \text{sign } z_1 + u \\ \dot{z}_2 &= z_1 \\ \alpha\dot{z}_3 &= \text{abs } (z_1) - \frac{z_3}{Q} \end{aligned} \tag{2.2}$$

where, abusing the notation, the symbol: " $\cdot$ " now represents derivation with respect to the scaled time, τ. The variable, u, is the normalized control input, necessarily restricted to take values in the discrete set,

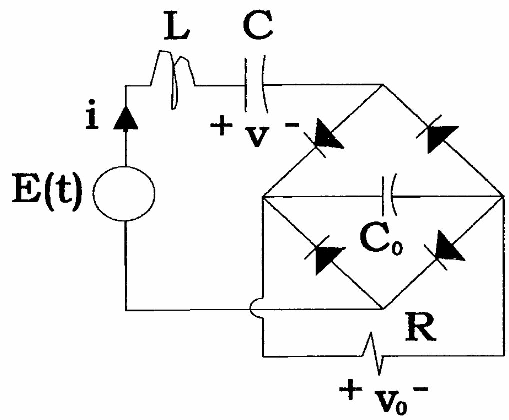

Figure 1: The series resonant converter.

$\{+1, -1\}$. The parameter Q, defined as $Q = R\sqrt{C/L}$, is known as the " quality factor" of the circuit, while the constant, α, is just the ratio, $\alpha = C_0/C$.

The normalized resonant converter may then be represented as the *hybrid automaton* shown in Figure 2.

2.1 Differential flatness of the hybrid converter

We propose to view the normalized converter system dynamics (2.2) as constituted by a "hybrid" combination of two linear controllable (i.e., differentially flat) systems, each one characterized by a corresponding flat output. Consider then the following pair of controllable linear systems, derivable from the system model for the instances in which $z_1 > 0$ and $z_1 < 0$, respectively.

$$
\begin{array}{l}
\text{for } z_1 > 0 \\
\begin{aligned}
\dot{z}_1 &= -z_2 - z_3 + u \\
\dot{z}_2 &= z_1 \\
\alpha\dot{z}_3 &= z_1 - \frac{z_3}{Q}
\end{aligned} \\
\text{for } z_1 < 0 \\
\begin{aligned}
\dot{z}_1 &= -z_2 + z_3 + u \\
\dot{z}_2 &= z_1 \\
\alpha\dot{z}_3 &= -z_1 - \frac{z_3}{Q}
\end{aligned}
\end{array}
\qquad (2.3)
$$

Indeed, on each state space *location* the system is constituted by a controllable and, hence, differentially flat system. As a result, there exists, in each case, a flat output y which is a linear combination of the state variables. Such outputs allow for a complete differential parameterization of each local representation of the system. The flat output variables are given by,

$$
\left\{
\begin{array}{lll}
y = z_2 - \alpha z_3 & ; & \text{for } z_1 > 0 \\
y = z_2 + \alpha z_3 & ; & \text{for } z_1 < 0
\end{array}
\right.
$$

which have the physical meaning, respectively, of being proportional to the difference and the sum of the instantaneous stored charges in the series capacitor, C, and the output capacitor, C_0.

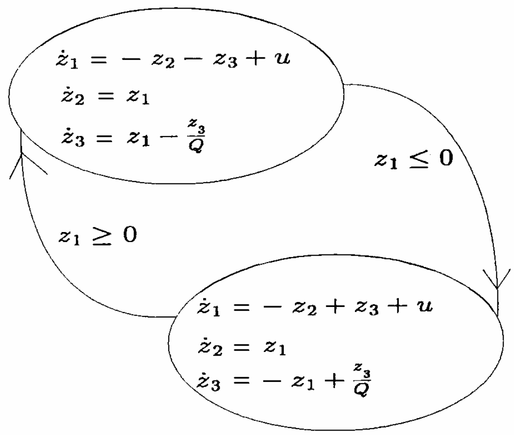

Figure 2: The normalized resonant converter as a hybrid automaton.

One readily obtains the following differential parameterization of the constitutive system variables in each case,

for $z_1 > 0$

$$\begin{aligned} z_3 &= Q\dot{y} \\ z_2 &= y + \alpha Q\dot{y} \\ z_1 &= \dot{y} + \alpha Q\ddot{y} \\ u &= \alpha Q y^{(3)} + \ddot{y} + Q(1+\alpha)\dot{y} + y \end{aligned}$$

for $z_1 < 0$

$$\begin{aligned} z_3 &= -Q\dot{y} \\ z_2 &= y + \alpha Q\dot{y} \\ z_1 &= \dot{y} + \alpha Q\ddot{y} \\ u &= \alpha Q y^{(3)} + \ddot{y} + Q(1+\alpha)\dot{y} + y \end{aligned}$$

The key observations, on which our control approach is based, are the following:

- The differential parameterizations associated with the flat outputs lead to the *same* differential relation between the flat output, y, and the control input u. In other words, independently of the region of the state space of the underlying hybrid system, the flat output satisfies the dynamics,

$$\alpha Q y^{(3)} + \ddot{y} + Q(1+\alpha)\dot{y} + y = u \qquad (2.4)$$

- The normalized series capacitor voltage, z_2, and the normalized inductor current, z_1, (i.e. the *resonant variables*) also exhibit the *same* parameterizations in terms of the corresponding flat output.

$$z_2 = y + \alpha Q\dot{y}, \quad z_1 = \dot{y} + \alpha Q\ddot{y}$$

  These representations are, therefore, invariant with respect to the structural changes undergone by the system.

- The input-to-flat output differential relation has a naturally observable realization from the only measured output.

Based on these observations, an output switching feedback law is synthesized. Simulation, as well as experimental, results of the feedback regulation of the studied converter are reported in the full version of the article.

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