SYNCHRONIZATION OF TWO HYPERCHAOTIC CHUA CIRCUITS: A GENERALIZED HAMILTONIAN SYSTEMS APPROACH [1]

César Cruz-Hernández [a,2] Cornelio Posadas-Castillo [a]
Hebert Sira-Ramírez [aa]

[a] Electronics and Telecommunications Department,
Scienti...c Research and Advanced Studies of Ensenada
(CICESE),
Km. 107, Carretera Tijuana-Ensenada, Ensenada, B. C., 22860,
México
[aa] Electrical Engineering Department,
Mechatronics Section
CINVESTAV-IPN, México

Abstract: In this paper, we use a Generalized Hamiltonian systems approach to synchronize two unidirectionally coupled hyperchaotic Chua circuits. Synchronization is thus between the transmitter and the receiver dynamics with the receiver being given by an observer. We apply this approach to transmit digital information signals in which the quality of the recovered signal is higher than in traditional observer techniques while the encoding remains potentially secure.Copyrigth ⓣ IFAC 2002

Keywords: Synchronization, hyperchaotic Chua's circuit, passivity based observers, secret communication.

1. INTRODUCTION

The application of chaotic synchronization to secure communication systems was suggested in an earlier article by (Pecora and Carroll 1990). Ever since the appearance of that paper, synchronization of chaotic systems has received a lot of attention and interest. Diferent approaches are being currently proposed and pursued (see, for instance, e.g., (Kocarev et al., 1992; Ogorzalek, 1993; Wu and Chua, 1993; Ding and Ott, 1994; Feldmann, et al., 1996; Nijmeijer and Mareels, 1997; Special Issue, 1997; Cruz and Nijmeijer, 2000; Sira-Ramírez

and Cruz, 2001; Pikovsky et al., 2001) and the references therein).

Data encryption using chaotic dynamics was reported in the early 1990s as a new approach for signal encoding which diꟲers from the conventional methods using numerical algorithms as the encryption key. As a result, the synchronization of chaotic systems plays an important role in the area of chaotic communications. The issue of security, however, naturally arises in the consideration of chaotic communication and it constitutes an important motivation for chaotic communication research. In particular, several techniques, such as chaotic masking (Cuomo et al., 1993), chaotic switching (Parlitz et al., 1992; Cuomo et al., 1993; Dedieu et al., 1993) and chaotic parameter modulation (Yang and Chua, 1996) have been developed. However, subsequent works have shown that these techniques have a low degree of security (see

(Short, 19994, 1996; Perez and Cerdeira, 1995; Yang, 1995)).

Two factors that are of primordial importance in security considerations related to chaotic communication systems. These are; the dimensionality of the chaotic attractor and the effort required to obtain the necessary parameters for the matching of a receiver dynamics.

One way to enhance the level of security of the communication system can consist in applying proper cryptographic techniques to the information signal (Yang et al., 1997). Another way to solve this security problem is to encode the message by using high dimensional chaotic attractors, or hyperchaotic atractrors, which take advantage of the increased randomness and unpredictability of the higher dimensional dynamics. In such option one generally encounters multiple positive Lyapunov exponents. However, the synchronization of hyperchaotic systems is a much more dif-...cult problem (Brucoli et al., 1996; Peng et al., 1996).

The objective of this paper is to extend the approach developed in (Sira-Ramírez and Cruz, 2001) to the synchronization of hyperchaotic circuits through a Generalized Hamiltonian systems approach. In this work we show that the synchronization of two hyperchaotic Chua circuits is possible from this viewpoint and, morevoer, we proceed to apply this approach to transmit digital information signals using chaotic parameter switching. We can enumerate several advantages over the existing methods:

² it enables synchronization be achieved in a systematic way; ·

² it can be successfully applied to several well-known hyperchaotic oscillators (e.g., Rössler's hyperchaotic system);

² it does not require the computation of any Lyapunov exponent;

² it does not require initial conditions belonging to the same basin of attraction.

2. SYNCHRONIZATION OF HYPERCHAOTIC CHUA'S CIRCUITS

Consider the following n-dimensional autonomous system

$$\underline{x} = f(x); \qquad x \; 2 \; R^n \qquad (1)$$

which represents a circuit exhibiting a hyperchaotic behavior. Following the approach provided in (Sira-Ramírez and Cruz, 2001), many physical systems described by Eq. (1) can be written in the following "Generalized Hamiltonian " Canonical form,

$$\underline{x} = J(x)\frac{@H}{@x} + S(x)\frac{@H}{@x} + F(x); \qquad (2)$$

where H (x) denotes a smooth energy function which is globally positive de...nite in R^n. The column gradient vector of H, denoted by @H=@x, is assumed to exist everywhere. We frequently use quadratic energy function H (x) = 1=2 x^T M x with M being a, constant, symmetric positive de...nite matrix. In such case, @H=@x = M x. The square matrices, J (x) and S (x) satisfy, for all x 2 R^n, the following properties, which clearly depict the energy managing structure of the system, J (x) + J^T (x) = 0 and S (x) = S^T (x). The vector ...eld J (x) @H=@x exhibits the conservative part of the system and it is also referred to as the workless part, or work-less forces of the system; and S (x) depicting the working or nonconservative part of the system. For certain systems, S (x) is negative de...nite or negative semide...nite. In such cases, the vector ...eld is addressed to as the dissipative part of the system. If, on the other hand, S (x) is positive de...nite, positive semidefinite, or inde...nite, it clearly represents, respectively, the global, semi-global and local destabilizing part of the system. In the last case, we can always (although nonuniquely) descompose such an inde...nite symmetric matrix into the sum of a symmetric negative semide...nite matrix R (x) and a symmetric positive semide...nite matrix N (x). And where F (x) represents a locally destabilizing vector ...eld.

We consider a special class of Generalized Hamiltonian systems with destabilizing vector ...eld and linear output map , y, given by

$$\underline{x} = J(y)\frac{@H}{@x} + (I + S)\frac{@H}{@x} + F(y); \qquad x \; 2 \; R^n$$

$$y = C\frac{@H}{@x}; \qquad y \; 2 \; R^m \qquad (3)$$

where S is a constant symmetric matrix, not necessarily of de...nite sign. The matrix I is a constant skew symmetric matrix. The vector variable y is referred to as the system output. The matrix C is a constant matrix.

We denote the estimate of the state vector x by », and consider the Hamiltonian energy function H (») to be the particularization of H in terms of ». Similarly, we denote by the estimated output, computed in terms of the estimated state ». The gradient vector @H(»)=@» is, naturally, of the form M».

A dynamic nonlinear state observer for the system (3) is readily obtained as

$$» = J(y)\frac{@H}{@»} + (I + S)\frac{@H}{@»} + F(y) + K(y \; i \; \dot{\,})$$

$$= C \frac{@H}{@\gg} \tag{4}$$

where K is a constant matrix, known as the observer gain.

The state estimation error, de...ned as $e = x_i \gg$ and the output estimation error, de...ned as $e_y = y_i$ ´, are governed by

$$\dot{e} = J(y)\frac{@H}{@e} + (I + S_i KC)\frac{@H}{@e}; \quad e \, 2 \, R^n$$

$$e_y = C\frac{@H}{@e}; \quad e_y \, 2 \, R^m \tag{5}$$

where the vector, $@H=@e$ actually stands, with some abuse of notation, for the gradient vector of the modi...ed energy function, $@H(e)=@e = @H=@x_i$ $@H=@\gg = M(x_i \gg) = Me$. Below, we set, when needed, $I + S = W$. We say that the receiver system (4) synchronizes with the transmitter system (3), if $e(t) ! 0$ as $t ! 1$.

Fig. 1. Pair of identical unidirectionally coupled Chua circuits.

Two coupled Chua circuits (see, (Anishchenko et al., 1994; Brucoli et al., 1996)). The hyper-chaotic circuit considered in this paper is formed by a pair of unidirectionally coupled identical Chua circuits as shown in Fig. 1. This circuit is described by

$$C_1\dot{x}_1 = G(x_2 _i x_1) _i F(x_1);$$
$$C_2\dot{x}_2 = G(x_1 _i x_2) + x_3 + C_2k(x_5 _i x_2);$$
$$L\dot{x}_3 = _i x_2; \tag{6}$$
$$C_1\dot{x}_4 = G(x_5 _i x_4) _i F(x_4);$$
$$C_2\dot{x}_5 = G(x_4 _i x_5) + x_6;$$
$$L\dot{x}_6 = _i x_5$$

where

$$F(x_1) = bx_1 + \frac{1}{2}(a_i b)(jx_1 + 1j_i jx_1 _i 1j); a; b < 0;$$

$$F(x_4) = bx_4 + \frac{1}{2}(a_i b)(jx_4 + 1j_i jx_4 _i 1j); a; b < 0:$$

The parameter k individualizes the unidirectional coupling between the two Chua circuits.

The set of di¤erential equations describing hyperchaotic Chua circuit in Hamiltonian Canonical form with a destabilizing ...eld is given by

Fig. 2. Double-double scroll attractor in hyper-chaotic Chua circuit with $k = 0.02$, $® = 10$, $= 14.87$, $a = _i 1.27$ and $b = _i 0.68$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & _i \frac{1}{C_2^2}k & \frac{1}{LC_2} & 0 & \frac{1}{2C_2^2}k & 0 \\ 0 & _i \frac{1}{LC_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & _i \frac{1}{2C_2^2}k & 0 & 0 & 0 & \frac{1}{LC_2} \\ 0 & 0 & 0 & 0 & _i \frac{1}{LC_2} & 0 \end{bmatrix} \frac{@H}{@x}$$

$$+ \begin{bmatrix} _i \frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\ \frac{G}{C_1C_2} & _i \frac{G}{C_2^2} & 0 & 0 & \frac{1}{2C_2^2}k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & _i \frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 \\ 0 & \frac{1}{2C_2^2}k & 0 & \frac{G}{C_1C_2} & _i \frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{@H}{@x}$$

$$+ \begin{bmatrix} _i \frac{1}{C_1}F(x_1) \\ 0 \\ 0 \\ _i \frac{1}{C_1}F(x_4) \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

taking as the Hamiltonian energy function the scalar function

$$H(x) = \frac{1}{2}£ C_1x_1^2 + C_2x_2^2 + Lx_3^2 + C_1x_4^2 + C_2x_5^2 + Lx_6^2 \,^\square \tag{8}$$

and gradient vector as

$$\frac{@H}{@x} = £ C_1x_1 \quad C_2x_2 \quad Lx_3 \quad C_1x_4 \quad C_2x_5 \quad Lx_6 \,^{\square T}$$

The destabilizing vector ...eld evidently calls for x_1 and x_4 to be used as the outputs, y_1 and y_2, of the transmitter (7). The matrices C, S and I, are given by

$$C = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0 \end{bmatrix};$$

$$S = \begin{bmatrix} i\dfrac{G}{C_1^2} & \dfrac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\[4pt] \dfrac{G}{C_1C_2} & i\dfrac{G}{C_2^2} & 0 & 0 & \dfrac{1}{2C_2}k & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & i\dfrac{G}{C_1^2} & \dfrac{G}{C_1C_2} & 0 \\[4pt] 0 & \dfrac{1}{2C_2}k & 0 & \dfrac{G}{C_1C_2} & i\dfrac{G}{C_2^2} & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & i\dfrac{1}{C_2}k & \dfrac{1}{LC_2} & 0 & \dfrac{1}{2C_2}k & 0 \\[4pt] 0 & i\dfrac{1}{LC_2} & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & i\dfrac{1}{2C_2}k & 0 & 0 & 0 & \dfrac{1}{LC_2} \\[4pt] 0 & 0 & 0 & 0 & i\dfrac{1}{LC_2} & 0 \end{bmatrix};$$

The pair $(C; S)$ is neither observable nor. detectable. However, the pair $(C; W)$ is observable. The system lacks damping in the x_3 and x_6 variables, and either in the x_1, x_2, x_4 or the x_5 variable as inferred from the negative semi-de...nite nature of the dissipation structure matrix, S. If x_1 and x_4 are used as outputs, then the outputs error injection terms can enhance the dissipation in the error state dynamics. The receiver is designed as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & i\dfrac{1}{C_2}k & \dfrac{1}{LC_2} & 0 & \dfrac{1}{2C_2}k & 0 \\[4pt] 0 & i\dfrac{1}{LC_2} & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & i\dfrac{1}{2C_2}k & 0 & 0 & 0 & \dfrac{1}{LC_2} \\[4pt] 0 & 0 & 0 & 0 & i\dfrac{1}{LC_2} & 0 \end{bmatrix} \dfrac{@H}{@\cdot}$$

$$+ \begin{bmatrix} i\dfrac{G}{C_1^2} & \dfrac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\[4pt] \dfrac{G}{C_1C_2} & i\dfrac{G}{C_2^2} & 0 & 0 & \dfrac{1}{2C_2}k & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & i\dfrac{G}{C_1^2} & \dfrac{G}{C_1C_2} & 0 \\[4pt] 0 & \dfrac{1}{2C_2}k & 0 & \dfrac{G}{C_1C_2} & i\dfrac{G}{C_2^2} & 0 \\[4pt] 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dfrac{@H}{@\cdot}$$

$$+ \begin{bmatrix} i\dfrac{1}{C_1}F(y_1) \\ 0 \\ 0 \\ i\dfrac{1}{C_1}F(y_2) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} y \qquad (9)$$

The choice of k_i, $i = 1; 2; :::; 6$ as arbitrary strictly positive constants su¢ces to guarantee the asymp-

totic exponential stability to zero of the synchronization error. The synchronization error dynamics is governed by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{2C_1}k_2 & \dfrac{1}{2C_1}k_3 & 0 & 0 & 0 \\[4pt] \dfrac{1}{2C_1}k_2 i & \dfrac{2}{C_2}k & \dfrac{2}{LC_2} & 0 & \dfrac{1}{C_2}k & 0 \\[4pt] \dfrac{1}{2C_1}k_3 i & \dfrac{2}{LC_2} & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & 0 & \dfrac{1}{2C_1}k_5 & \dfrac{1}{2C_1}k_6 \\[4pt] 0 & i\dfrac{1}{C_2}k & 0 & i\dfrac{1}{2C_1}k_5 & 0 & \dfrac{2}{LC_2} \\[4pt] 0 & 0 & 0 & i\dfrac{1}{2C_1}k_6 i & \dfrac{2}{LC_2} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \bar{A} & \acute{A} & 0 & 0 & 0 \\[4pt] \acute{A} & i\dfrac{G}{C_2^2} & 0 & 0 & \dfrac{1}{2C_2}k & 0 \\[4pt] \acute{A} & 0 & 0 & 0 & 0 & 0 \\[4pt] 0 & 0 & 0 & 1 & \# & \tfrac{3}{4} \\[4pt] 0 & \dfrac{1}{2C_2}k & 0 & \# & i\dfrac{G}{C_2^2} & 0 \\[4pt] 0 & 0 & 0 & \tfrac{3}{4} & 0 & 0 \end{bmatrix} \dfrac{@H}{@e} \qquad (10)$$

where

$$\iota = i\,\dfrac{G + k_1C_1}{C_1^2}; \quad \bar{A} = i\,\dfrac{2G i\, k_2C_2}{2C_1C_2}; \quad \acute{A} = i\,\dfrac{1}{2C_1}k_3$$

$$\imath = i\,\dfrac{G + k_4C_1}{C_1^2}; \quad \# = i\,\dfrac{2G i\, k_5C_2}{2C_1C_2}; \quad \tfrac{3}{4} = i\,\dfrac{1}{2C_1}k_6$$

To ease the simulations we resorted the following normalized version of the hyperchaotic circuit (see (Anishchenko et al., 1994))

$$\dot{x}_1 = \circledR (x_2 i\, x_1 i\, f(x_1));$$
$$\dot{x}_2 = x_1 i\, x_2 + x_3 + k(x_5 i\, x_2);$$
$$\dot{x}_3 = i\, \bar{\;}x_2; \qquad (11)$$
$$\dot{x}_4 = \circledR (x_5 i\, x_4 i\, f(x_4));$$
$$\dot{x}_5 = x_4 i\, x_5 + x_6;$$
$$\dot{x}_6 = i\, \bar{\;}x_5$$

where

$$f(x_1) = bx_1 + \dfrac{1}{2}(a i\, b)(jx_1 + 1j i\, jx_1 i\, 1j);$$

$$f(x_4) = bx_4 + \dfrac{1}{2}(a i\, b)(jx_4 + 1j i\, jx_4 i\, 1j):$$

The transmitter and the receiver are both characterized by the following parameter values: $\circledR = 10$, $\bar{\;} = 14{:}87$, $a = i\, 1{:}27$, $b = i\, 0{:}68$ and $k = 0{:}02$; and by the initial conditions:

$$x(0) = (0{:}01; 0{:}01; 0{:}01; 0{:}011; 0{:}01; 0{:}01);$$

$$\hat{x}(0) = (0{:}011; 0; 0; 0{:}012; 0; 0):$$

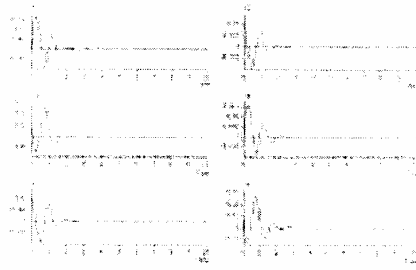These values assure the existence of the system hyperchaotic behavior. Figure 2 shows the double-

Fig. 3. Synchronization error evolution $e_i = x_i$ $\i$
$\gg_i$, i = 1; 2; :::; 6 between (9) and (11).

double scroll attractor in hyperchaotic Chua circuit (11). While Figure 3 shows the synchronization error evolution ($e_i = x_i$ $\i$ $\gg_i$, i = 1; 2; :::; 6) between transmitter system (7) and receiver system (9) for the observer gain $k_1 = 2$, $k_2 = 3$, $k_3 = 0$, $k_4 = 2$, $k_5 = 3$, and $k_6 = 0$. We can see, after some transient behavior, that all the hyperchaotic states of transmitter (7) synchronize with the corresponding states of receiver (9).

3. SYNCHRONIZATION STABILITY ANALYSIS

In this section, we examine the stability of the synchronization error (10) between hyperchaotic Chua circuit (7) and observer (9). A necessary and su¢cient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

Theorem 1. (Sira-Ramírez and Cruz, 2001). The state x of the nonlinear system (7) can be globally, exponentially, asymptotically estimated, by the state $\gg$ of the observer (9) if and only if there exists a constant matrix K such that the symmetric matrix

$$[W \i KC] + [W \i KC]^T = [S \i KC] + [S \i KC]^T$$
$$= 2 S_i \frac{1}{2}\i KC + C^T K^{T^\mathbb{C}}$$

is negative de...nite.

4. APPLICATION TO CHAOTIC COMMUNICATIONS

Here, we apply the Hamiltonian synchronization of hyperchaotic Chua circuits to chaotic switching. In this technique, the message m (t) is a binary signal, and is used to modulate one or more parameters of the transmitter, i.e. m (t) controls a switch whose action changes the parameter values of the transmitter. Thus, according to the value of m (t) at any given time t, the transmitter has
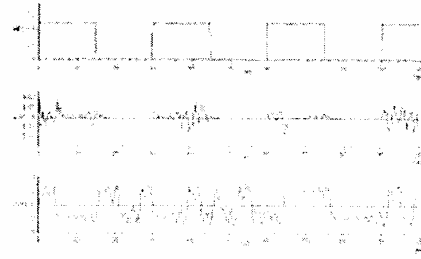

Fig. 4. (a) Digital information signal m (t), (b) synchronization error detection e_1 (t), (c) transmitted hyperchaotic signal x_1 (t).

either the parameter set value p or the parameter set value p̂. At the receiver, m (t) is decoded by using the synchronization error to decide whether the received signal corresponds to one parameter value, or the other (it can be interpreted as an one or zero). In our case, to transmit m (t) via chaotic switching, let ¯ be the parameter to be modulated in the hyperchaotic Chua transmitter (11), the parameter ® was ...xed. We use a "modulation rule" to modulate m (t) as follow ¯ (t) = ¯ + r t m (t), where r = 0:2 while the message is de...ned as

$$m(t) = \begin{cases} 1; & 0 \cdot t < 15; \\ 0; & 15 \cdot t < 30; \\ 1; & 30 \cdot t < 45; \\ 0; & 45 \cdot t < 60; \\ 1; & 60 \cdot t < 75; \\ 0; & 75 \cdot t < 90; \end{cases}$$

The following ...gures illustrate the binary transmission of m (t) when ¯ is switched between ¯ (1) = 14:87 and ¯ (0) = 15:07. Figure 4 depicts: (a) the message to be transmitted, (b) the synchronization error detection e_1 (t) and (c) the transmitted hyperchaotic signal x_1 (t); whereas, Figure 5 shows: (a) the message to be transmitted, (b) the synchronization error detection e_4 (t) and (c) the transmitted hyperchaotic signal x_4 (t). Figures 4(b) and 5(b) clearly show that the original message is recovered at the receiver.

5. CONCLUDING REMARKS

In this paper, we have approached the problem of synchronization of two hyperchaotic Chua circuits from the perspective of Generalized Hamiltonian systems including dissipation and destabilizing terms. The approach allows one to give a simple design procedure for the receiver system and clari...es the issue of deciding on the nature of the output signal to be transmitted. Our scheme can be successfully applied to several well-known hyperchaotic osillators (e.g., Rössler's
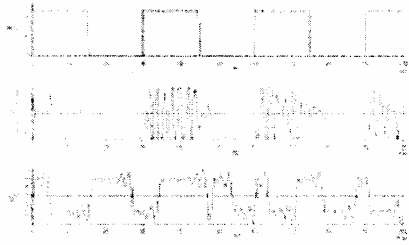
Fig. 5. (a) Digital information signal $m(t)$, (b) synchronization error detection $e_4(t)$, (c) transmitted hyperchaotic signal $x_4(t)$.

hyperchaotic system with a scalar transmitted signal). The approach is easily implemented on experimental setups, and shows great potential for actual communication systems in which the encoding is required to be secure.

In a forthcomming article we will be concerned with a physical implementation of the method with a speci...c quantization of the degree of safety of the proposal in actual communication systems.

REFERENCES

Anishchenko, V.S., Kapitaniak, T., Safonova, M.A. and Sosnovzeva, O.V. (1994). Birth of double-double scroll attractor in coupled Chua circuits, Phys. Lett. A192, 207-214.

Brucoli, M., Carnimeo, L. and Grassi, G. (1996). A method for the synchronization of hyperchaotic circuits, Int. J. Bifurc. Chaos 6(9), 1673-1681.

Cruz, C. and Nijmeijer, H. (2000). Synchronization through ...ltering, Int. J. Bifurc. Chaos 10(4), 763-775.

Cuomo, K.M., Oppenheim, A.V. and Strogratz, S.H. (1993). Synchronization of Lorenz-based chaotic circuits with applications to communications, IEEE Trans. Circuits Syst. II 40(10), 626-633.

Dedieu, H., Kennedy, M. and Hasler, M. (1993). Chaotic shift keying: Modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuits, IEEE Trans. Circuits Syst. II 40(10), 634-642.

Ding, M. and Ott, E. (1994). Enhancing synchronism of chaotic systems, Physical Review E, 49(2), 945-948.

Feldmann, U., Hasler, M. and Schwarz, W. (1996). Communication by chaotic signals: the inverse system aproach, Int. J. Circuits Theory and Applications, 24, 551-579.

Kocarev, Lj., Halle, K.S., Eckert, K. and Chua, L.O. (1992). Experimental demostration of secure communications via chaotic synchronization, Int. J. Bifurc. Chaos 2(3), 709-713.

Nijmeijer, H. and Mareels, I.M.Y. (1997). An observer looks at synchronization, IEEE Trans. Circuits Syst. I 44(10), 882-890.

Ogorzalek, M.J. (1993). Taming Chaos-Part I: Synchronization, IEEE Trans. on Circuits and Systems-I, 40(10), 693-699.

Parlitz, U., Chua, L.O., Kocarev, Lj., Halle, K.S. and Shang, A. (1992). Transmission of digital signals by chaotic synchronization, Int. J. Bifurc. Chaos 2(4), 973-977.

Pecora, L.M. and Carroll, T.L. (1990). Synchronization in chaotic systems, Phys. Rev. Lett. 64, 821-824.

Peng, J.H., Ding, E.J., Ding, M. and Yang, W. (1996). Synchronizing hyperchaos with a scalar transmitted signal, Phys. Rev. Lett. 76, 904-907.

Perez, G. and Cerdeira, H.A. (1995). Extracting messages masked by chaos, Phys. Rev. Lett. 74, 1970-1973.

Pikovsky, A., Rosenblum, M. and Kurths, J. (2001). Synchronization: A Universal Concept in Nonlinear Sciences, Cambridge University Press.

Short, K.M. (1994). Steps towards unmasking chaotic communication, Int. J. Bifurc. Chaos 4(4), 959-977.

Short, K.M. (1996). Unmasking a modulated chaotic communications scheme, Int. J. Bifurc. Chaos 6(2), 367-375.

Sira-Ramírez, H. and Cruz, C. (2001). Synchronization of Chaotc Systems: A Generalized Hamiltonian Systems Approach, Int. J. Bifurc. Chaos 11(5), 1381-1395. And in Procs. of American Control Conference (ACC' 2000), Chicago, USA, 769-773.

Special Issue (1997). Chaos synchronization and control: Theory and applications, IEEE Trans. Circuits Syst. I 44(10).

Yang, T. (1995). Recovery of digital signals from chaotic switching, Int. J. Circuit Theory and Applications, 23, 611-615.

Yang, T. and Chua, L.O. (1996). Secure communication via chaotic parameter modulation, IEEE Trans. Circuits Syst. I 43(9), 817-819.

Yang, T., Wu, C.W. and Chua, L.O. (1997). Cryptography based on chaotic system, IEEE Trans. Circuits Syst. I 44(5), 469-472.

Wu, C.W. and Chua, L.O. (1993). A simple way to synchronize chaotic systems with applications to secure communication systems, Int. J. of Bifurc. Chaos, 3(6), 1619-1627.