# Passivity-based Control of Euler-Lagrange Systems:

## Mechanical, Electrical and Electromechanical Applications

**Romeo Ortega,**
**Antonio Loría,**
**Per J. Nicklasson,**
**Hebertt Sira-Ramírez.**

*To Amparo with all my love,*
Romeo.

*To Lena with* $r = 1 - sin\theta$*,*
*to Mari my sister with my deepest admiration,*
Toño.

*To my parents,*
Per Johan.

*To José Humberto Ocariz E. with respect and affection,*
*to María Elena and María Gabriela with all my love,*
Hebertt.

# Preface

By its own definition the final purpose of control is to *control* something. In fact, the foundational developments of Huygens, Maxwell, Routh, Minorsky, Nyquist and Black (to name a few) were motivated by real–world applications. In the hands of mathematicians such as Wiener, Bellman, Lefschetz, Kalman and Pontryagin (again, to name just a few) control theory developed in the 1950s and 1960s as a branch of applied mathematics, independent of its potential application to engineering problems. Some tenuous arguments were typically invoked to provide some practical motivation to the research on this so–called mathematical control theory. For instance, the study of the triple $(A,\ B,\ C)$ was rationalized as the study of the linearization of an arbitrary nonlinear system –an argument that had a grain of truth. By the end of the 1980s a fairly complete body of knowledge for general linear systems –including powerful techniques of controller synthesis– had been completed. Some spectacular applications of this theory to practical situations that fitted the linear systems paradigm were reported.

The attempt to mimic the developments of linear systems theory in the general nonlinear case enticed many researchers. Extensions to a fairly general class of nonlinear systems of the basic concepts of controllability, observability, and realizability were crowned with great success. The controller synthesis problem proved to be, however, much more elusive. Despite some significant progress, to date, general techniques for stabilization of nonlinear systems are available only for special classes of nonlinear systems. This is, of course, due to the daunting complexity of the behaviour of nonlinear dynamic systems which puts a serious question mark on the interest of aiming at a monolithic synthesis theory. On the other hand, new technological developments had created engineering problems where certain well–defined nonlinear effects had to be taken into account. Unfortunately, the theory developed for general nonlinear systems could not successfully deal with them, basically because the "admissible structures" were determined by analytical considerations, which do not necessarily match the physical constraints. It became apparent that to solve these new problems, the "find an application for my theory" approach had to be abandoned, and a new theory tailored for the application had to be worked out.

The material reported in this book is an attempt in this direction. Namely, we start from a well–defined class of systems to be controlled and try to develop a theory best suited for them. As the title suggests, the class we consider covers a very broad spectrum (Euler–Lagrange systems with mechanical, electrical and electromechanical applications) however, detailed analysis is presented only for robots, AC machines and power converters. We have found that this set of applications is sufficiently general —it has at least kept us busy for the last 10 years!

Different considerations and techniques are used to solve the various problems however, in all cases we strongly rely on the information provided by the variational modeling and, in particular, concentrate our attention on the energy and dissipation functions that define the dynamics of the system. A second unifying thread to all the applications is the fundamental concept of passivity. Finally, a recurrent theme throughout our work is the notion of interconnection that appears, either in the form of a feedback decomposition instrumental for the developments, or as a framework for focusing on the relevant parts of a model.

An important feature of the proposed controller design approach is that it is based on the input–output property of passivity, hence it will typically not require the measurement of the full state to achieve the control objectives. Consequently, throughout the book we give particular emphasis to (more realistic, but far more challenging) output–feedback strategies.

The book is organized in the following way. In Chapter **1** we present first a brief introduction that explains the background of the book and elaborates upon its three keywords: Euler–Lagrange (EL) systems, passivity and applications. The notion of passivity–based control (PBC) is explained in detail also in this chapter, underscoring its conceptual advantages. The main background material pertaining to EL systems is introduced in Chapter **2**. In particular we mathematically describe the class of systems that we study throughout the book, exhibit some fundamental input–output and Lyapunov stability properties, as well as some basic features of their interconnection. We also give in this section the models of some examples of physical systems that will be considered in the book.

The remaining of the book is divided in three parts devoted to mechanical, electrical and electromechanical systems, respectively. The first part addresses a class of mechanical systems, of which a prototypical example are the robot manipulators, but it is not restricted to them. For instance, we consider also applications to simple models of marine vessels and rotational translational actuators. The results concerning mechanical systems are organized into set point regulation (Chapter **3**), trajectory tracking (Chapter **4**) and adaptive disturbance attenuation, with application to friction compensation (Chapter **5**). The theoretical results are illustrated with realistic simulation results. In this part, as well as in other sections of the book, we carry out comparative studies of the performance obtained by PBC with those achievable with other schemes. In particular, for robots with flexible joints, we compare in Chapter **4** PBC with schemes based on backstepping and cascaded systems.

The second part of the book is dedicated to electrical systems, in particular DC-

DC power converters. In Chapter **6** the EL model is derived and the control relevant properties are presented. We present both, a switched model that describes the exact behaviour of the system with a switching input, and an approximate model for the pulse width modulator controlled converters. While in the first model we have to deal with a hybrid system (with inputs 0 or 1), in the latter model, which is valid for sufficiently high sampling frequencies, the control input is the duty ratio which is a continuous function ranging in the interval $[0,1]$. Besides the standard EL modeling, we also present a rather novel, and apparently more natural, Hamiltonian model that follows as a particular case of the extended Hamiltonian models proposed by Maschke and van der Schaft.

Chapter **7** is devoted to control of DC-DC power converters. We present, of course, PBC for the average models. To deal with hybrid models we also introduce the concept of PBC with sliding modes. We show that combining this two strategies we can reduce the energy consumption, a well–known important drawback of sliding mode control. Adaptive versions of these schemes, that estimate on–line the load resistance are also derived. An exhaustive experimental study, where various linear and nonlinear schemes are compared, is also presented.

In the third part of the book we consider electromechanical systems. To handle this more challenging problem we introduce a feedback decomposition of the system into passive subsystems. This decomposition naturally suggests a nested–loop controller structure, whose basic idea is presented in a motivating levitated system example in Chapter **8**. This simple example helps us also to clearly exhibit the connections between PBC, backstepping and feedback linearization. In Chapters **9**–**11** we carry out a detailed study of nonlinear control of AC motors. The torque tracking problem is first solved for the generalized machine model in Chapter **9**. As an off–spin of our analysis we obtain a systems invertibility interpretation of the well-known condition of Blondel–Park transformability of the machine.

The next two chapters, **10** and **11**, are devoted to voltage–fed and current–fed induction machines, respectively. For the voltage–fed case we present, besides the nested–loop scheme, a PBC with total energy shaping. Connections with the industry standard field oriented control and feedback linearization are thoroughly discussed. These connections are further explored for current–fed machines in Chapter **11**. First, we establish the fundamental result that, for this class of machines, PBC exactly reduces to field oriented control. Then, we prove theoretically and experimentally that PBC outperforms feedback linearization control. The robustness of PBC, as well as some simple tuning rules are also given. Finally, motivated by practical considerations, a globally stable discrete–time version of PBC is derived. Both chapters contain extensive experimental evidence.

At last, in Chapter **12** we study the problem of electromechanical systems with nonlinear mechanical dynamics. The motivating example for this study is the control of robots with AC drives, for which we give a complete theoretical answer. The chapter clearly illustrates how PBC, as applied to Euler–Lagrange models, yields a modular design which effectively exploits the features of the interconnections. In a simulation study we compare our PBC with a backstepping design showing, once again, the superiority of PBC.

Background material on passivity, variational modeling and vector calculus are included in Appendices **A**, **B** and **C**, respectively.

The book is primarily aimed at graduate students and researchers in control theory who are interested in engineering applications. It contains, however, new theoretical results whose interest goes beyond the specific applications, therefore it might be useful also to more theoretically oriented readers. The book is written with the conviction that to deal with modern engineering applications, control has to reevaluate its role as a component of an interdisciplinary endeavor. A lot of emphasis is consequently given to modeling aspects, analysis of current engineering practice and experimental work. For these reasons it may be also of interest for students and researchers, as well as practitioning engineers, involved in more practical aspects of robotics, power electronics and motor control. For this audience the book may provide a source to enhance their theoretical understanding of some well–known concepts and to establish bridges with modern control theoretic concepts.

We have adopted the format of theorem–proof–remark, which may give the erroneous impression that it is a "theoretical" book, this is done only for ease of presentation. Although most of the results in this book are new, they are presented at a level accessible to audiences with a standard undergraduate background in control theory and a basic understanding of nonlinear systems theory. In order to favour the "readability" of our book we have moved some of the most "technical" proofs to Appendix **D**.

The material contained in the book summarizes the experience of the authors on control engineering applications over the last 10 years. It builds upon the PhD theses of the second and fourth author as well as collaborative research among all of us, and with several other researchers. Numerous colleagues and collaborators contributed directly and indirectly, and in various ways to this book.

The first author is particularly indebted to his former PhD students: G. Espinosa and R. Kelly triggered his interest in the areas of electrical machines and robotics, respectively, we have since kept an intensive and very productive research collaboration; G. Escobar, K. Kim and D. Taoutaou carried out some of the experimental work on converters and electrical machines. He has also enjoyed a long scientific collaboration with L. Praly who always provided insightful remarks and motivation to his work. Many useful scientific exchanges have been carried out over the years with H. Nijmeijer, M. Spong and A. J. van der Schaft, while Henk and Arjan motivated him to improve his theoretical background, Mark always found the threshold necessary to make a robot turn. He would like to thank all his co–authors from whom he learned the importance of collaborative work. Finally, he wants to express his deep gratitude to the french CNRS, which provides his researchers with working conditions unparalleled by any other institution in the world.

The second author wishes to acknowledge specially the collaboration with his former undergraduate-school teacher R. Kelly who earlier introduced him to robot control and Lyapunov theory. The enthusiasm of Rafael on these topics increased the motivation of the second author to pursue a doctoral degree in the field. During his doctoral research period he was also enriched with the advice and collaboration of H. Nijmeijer and L. Praly. The author wishes to express as well his deepest gratitude to his fiancee and collaborator E. Panteley for her fundamental moral support in this project and for helping with the figures of Chapter **6**. Last but not least, the work of the second author has been sponsored by the institutions he has been affiliated to in the past 5 years, in chronological order: CONACyT, Mexico; University of Twente, The Netherlands; University of Trondheim, Norway; and University of California at Santa Barbara, USA.

The third author wants to thank Research Director Peter Singstad, SINTEF Electronics and Cybernetics, Automatic Control, for supporting parts of this project financially.

The fourth author is indebted to his colleagues and students of the Control Systems Department of the Universidad de Los Andes (ULA) in Mérida (Venezuela) for the continuous support over the years in many academic endeavors. Special thanks and recognition are due to his former student, Dr. Orestes Llanes-Santiago, for his creative enthusiasm and hard work in the area of switched power converters. Visits to R. Ortega, since 1995, have been generously funded by the Programme de Cooperation Postgradué (PCP), by the National Council for Scientific Research of Venezuela (CONICIT), as well as by the Centre National de la Recherche Scientifique (CNRS) of France. Thanks are due to Professor Marisol Delgado, of the Universidad Simón Bolívar, who has acted as a highly efficient PCP Coordinator in Venezuela. Over the years, the author has benefited from countless motivational discussions with his friend Professor Michel Fliess of the Laboratoire des Signaux et Systèmes (CNRS), France. His experience and vision has been decisively helpful in many of the author's research undertakings.

R. Ortega, A. Loría, P. J. Nicklasson, H. Sira-Ramírez,
May 1998.

# Contents